ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
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Magnetoelastic Vibrations, Localized in the Vicinity of the Free Edge of a Thin Plate

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Բարակապատ սալի ազատ եզրի շրջակայքում մագնիսաառաձգական տեղայնացված տատանումներ

Կիրխոֆի տեսության և իդեալական հաղորդիչ միջավայրի մոդելի հիման վրա դիտարկվում է սալի մագնիսաառաձգական տատանումների խնդիրը: Հաստատված են ազատ եզրի շրջակայքում տեղայնացված լայնական տատանումների գոյության պայմանները: Գտնված է, որ մագնիսական դաշտի միջոցով կարելի է վերացնել տեղայնացված տատանումները:

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Магнитоупругие колебания, локализованные в окрестности свободного края тонкой пластинки.

На основе теории пластин Кирхгофа и модели идеально проводящей среды рассматривается задача магнитоупругих колебаний пластинки в продольном магнитном поле. Установлены условия существования локализованных изгибных колебаний у свободного края пластинки. Показано, что при помощи магнитного поля можно устранить локализованные колебания.

We will consider the problem of magnetoelastic vibrations of plate in the longitudinal magnetic field based on the Kirchhoff plate theory and the model of perfect conductive medium. The conditions of the existence of localized bending vibrations in the vicinity of the free edge of the plate have been established. It was shown that the localized vibrations could be eliminated by means of the magnetic field.

Y.K. Konenkov [1] was the first who investigated the problem of localized bending plate vibrations. At a later time this direction was widely developed as we can judge by the literature mentioned in [2-4].

As a basis for research of vibrations in an electro conductive plate served simple models – the model of the perfect conductive medium [5] and the model of the “weak” conductive medium [6]. Later the hypothesis of magnetoelasticity of thin bodies [7, 8] was suggested and allowed us to reduce the spatial problems of magentoelastic vibrations to two-dimensional ones. The researches on problems of magneto elastic vibrations of the plate on the basis of exact solutions and the hypothesis of magnetoelasticity of thin bodies have shown that the application of simple models depends on the configuration of the external magnetic field (longitudinal or transversal) as well as on the character of the considered problem (planar or transversal vibrations) [9, 10]. In particular, it was determined that the model of perfect conductive medium can be applied (with the adequate accuracy) to the problem of plate bending vibrations at presence of the longitudinal magnetic field but it could not be applied to the problem of planar vibrations in the same magnetic field [9, 11, 12].

1. The plate in the Cartesian frame of reference holds the position: $0 \le x \le a$, $0 \le y \le b$, $-h \le z \le h$. The plate in the unperturbed condition is situated in the constant magnetic field parallel to Ox axis.

$$\vec{H}_0 = H_0 \vec{i}, \quad H_0 = \text{const} \tag{1.1}$$

Let us assume that the plate is isotropic, homogeneous and perfect conductive. In this case instead of perfect relationship for the plate perturbed condition

$$\vec{j} = \sigma\left(\vec{e} + \frac{\mu}{c}\frac{\partial \vec{u}}{\partial t} \times \vec{H}_0\right) \tag{1.2}$$

We will take [5]

$$\vec{e} = -\frac{\mu}{c}\frac{\partial \vec{u}}{\partial t} \times \vec{H}_0 \tag{1.3}$$

Where $\vec{u}$ - is the elastic displacement vector, $\vec{j}$ - is the induced density of the electric current, $\vec{e}$ - is the perturbed electrical field, σ - is the electroconductivity coefficient, μ - is the magnetic permability of the plate material, c - is the constant equal to the velocity of light in vacuum (in the Gaussian system of units).

According to (1.3) model and electrodynamics equations, the perturbed magnetic field, the induced electric current and the body force exerting on the plate are defined in the following way

$$\vec{h} = \text{rot}\left(\vec{u} \times \vec{H}_0\right), \quad \vec{j} = \frac{c}{4\pi}\text{rot } \vec{h}, \quad \vec{R} = \frac{\mu}{c}\left(\vec{j} \times \vec{H}_0\right) \tag{1.4}$$

The equations of the plate vibrations have the following appearance:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + R_i = \rho \frac{\partial^2 u_i}{\partial t^2} \qquad (i = 1,2,3) \tag{1.5}$$

where u_i are the components of vector $\vec{u}$, R_i are the components of vector $\vec{R}$, σ_{ij} are the components of the stress tensor, ρ - is the density of the plate material. According to (1.4) for the considered problem the R_i components are defined in the following way:

$$R_1 = 0, \quad R_2 = \frac{\mu H_0^2}{4\pi}\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_3}{\partial y \partial z}\right)$$
$$R_3 = \frac{\mu H_0^2}{4\pi}\left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial z^2} + \frac{\partial^2 u_2}{\partial y \partial z}\right) \tag{1.6}$$

The following boundary conditions should be set on the plate's $z = \pm h$ face planes:

$$\sigma_{3i} + t_{3i} = t_{3i}^{(e)} \quad (i = 1,2,3) \tag{1.7}$$

where $t_{ij}, t_{ij}^{(e)}$ are the components of Maxwell symmetric linearized tensor, correspondingly inside the region occupied by the plate and outside of it. If we take into account the (1.1) conditions then the expressions for the components of Maxwell tensor look like:

$$t_{11} = \frac{\mu}{4\pi} H_0 h_1, \quad t_{22} = t_{33} = -\frac{\mu}{4\pi} H_0 h_1$$
$$t_{12} = \frac{\mu}{4\pi} H_0 h_2, \quad t_{13} = \frac{\mu}{4\pi} H_0 h_3, \; t_{23} = 0 \tag{1.8}$$

The components of $t_{ij}^{(e)}$ tensor for the media surrounding the plate are being defined in similar way with taking into account $\mu = 1$ condition.

It is clear from (1.7) boundary conditions that in general it is necessary to sovle the plate vibration equations jointly with the electrodynamics equations for the media surrounding the plate. This circumstance essentially complicates the researches on problems of magentoelastic vibrations of the plate. But we can overcome this obstacle for this type of problem when the magnetic field is parallel to the planes bounding the plate. Noting that the conditions of continuity of the normal components of perturbed induction of the magnetic field ($\mu h_3 = h_3^{(e)}$) should be satisfied on the $z = \pm h$ planes and neglecting the $t_{33}^{(e)}$ in comparison with t_{33} (because of the discontinuity of the tangential component of the perturbed magnetic field), the (1.7) boundary conditions are being replaced with the following ones:

$$\sigma_{31} = 0, \; \sigma_{32} = 0, \quad \sigma_{33} + t_{33} = 0 \tag{1.9}$$

The mentioned approximate (1.9) boundary conditions are based on the fact that during the resolving the mentioned problem with taking into account the Kirchhoff hypothesis for the infinite plate [11], the (1.7) and (1.9) boundary conditions give the same result for the plate's self-vibration frequency

2. According to the Kirchhoff plate theory we will take into account the following assumptions

the assumptions for displacements:

$$u_1 = u - z\frac{\partial w}{\partial x},\quad u_2 = v - z\frac{\partial w}{\partial y},\quad u_3 = w,\quad u,v,w \quad -(x,y,z) \tag{2.1}$$

the assumptions for main stresses

$$\sigma_{11} = \frac{E}{1-\nu^2}\left(\varepsilon_{11} + \nu\varepsilon_{22}\right),\quad \sigma_{22} = \frac{E}{1-\nu^2}\left(\varepsilon_{22} + \nu\varepsilon_{11}\right),\quad \sigma_{12} = \frac{E}{1+\nu}\varepsilon_{12} \tag{2.2}$$

The usual procedure of averaging the plate vibration equations (1.5) is being implemented with taking into consideration the (1.6) expressions, (1.9) boundary conditions, (2.1) assumptions and neglecting the moment of rotatory inertia. The averaged equations look like

$$\frac{\partial T_1}{\partial x} + \frac{\partial S}{\partial y} = 2\rho h\frac{\partial^2 u}{\partial t^2},\quad \frac{\partial S}{\partial x} + \frac{\partial T_2}{\partial y} + \frac{\mu h H_0^2}{2\pi}\left[\Delta u + L(u,v)\right] = 2\rho h\frac{\partial^2 v}{\partial t^2} \tag{2.3}$$

$$\begin{gathered} \frac{\partial N_1}{\partial x} + \frac{\partial N_2}{\partial y} + \frac{\mu h H_0^2}{2\pi}\frac{\partial^2 w}{\partial x^2} = 2\rho h\frac{\partial^2 w}{\partial t^2} \\ \frac{\partial M_1}{\partial x} + \frac{\partial H}{\partial y} = N_1;\quad \frac{\partial H}{\partial x} + \frac{\partial M_2}{\partial y} = N_2 \end{gathered} \tag{2.4}$$

We used the generally accepted designations [12] for stresses and moments in (2.3), (2.4) . The $L(u,v)$ operator is identically equal to zero if in expression for R_2 from (1.6) we accept, according to (2.1), that $\varepsilon_{33} = \partial u_3/\partial z = 0$.

$$L(u,v) = -\frac{\nu}{1-\nu}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{2.5}$$

and substitute ε_{33} from the Hooke's law into R_2 .

The substitution of the stresses and moments expressions into (2.3) and (2.4) equations brings to the following equations regarding the displacements of the plate's middle plane.

$$\begin{gathered} \Delta u + \vartheta\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{1}{c_t^2}\frac{\partial^2 u}{\partial t^2} \\ (1+\lambda)\Delta v + (\vartheta - \beta)\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{1}{c_t^2}\frac{\partial^2 v}{\partial t^2} \end{gathered} \tag{2.6}$$

$$D\Delta^2 w - \frac{\mu h H_0^2}{2\pi}\frac{\partial^2 w}{\partial x^2} + 2\rho h\frac{\partial^2 w}{\partial t^2} = 0 \tag{2.7}$$

where

$$\vartheta = \frac{1+\nu}{1-\nu},\quad \lambda = \frac{\mu H_0^2}{4\pi G},\quad \beta = \frac{\nu\lambda}{1-\nu}\quad \text{in view of } \varepsilon_{33} \neq 0 \tag{2.8}$$

As it is in Kirchhoff regular plate theory, the equations defining the planar and bending vibrations are being separated. The (2.7) bending magnetoelastic vibrations equation is the analogous to the bending vibrations equation of the plate. pre-extended in x direction.

3. The boundary conditions on the plate's $x = \text{const}$ and $y = \text{const}$ edges are obtained via the averaging the boundary conditions of the spatial problem of the theory of magnetoelasticity of the perfect conductive medium. It is evident that the boundary conditions for the fixed edge are the same as in the regular plate theory. In case of Navier conditions establishing on the $x = \text{const}$ edge

$$\sigma_{11} + t_{11} = t_{11}^{(e)}, \quad u_2 = 0, \quad u_3 = 0 \tag{3.1}$$

it is taken into account that the t_{11} component is continuous, because $\mu h_1 = h_1^{(e)}$. Then the (3.1) conditions averaging leads to the regular conditions for the hinge joint.

$$T_1 = 0, \quad v = 0, \quad w = 0, \quad M_1 = 0 \tag{3.2}$$

Let us assume that the conditions of sliding contact are established on the $x = \text{const}$ edge.

$$u_1 = 0, \quad \sigma_{12} + t_{12} = t_{12}^{(e)}, \quad \sigma_{13} + t_{13} = t_{13}^{(e)} \tag{3.3}$$

The $t_{12}^{(e)}, t_{13}^{(e)}$ terms in (3.3) are neglected in comparision with correspondingly t_{12}, t_{13} and then the averaging by thickness of the plate is implemented. Fianlly, the conditions of sliding contact are obtained in the following way:

$$u = 0, \quad S + \frac{\mu h H_0^2}{2\pi}\frac{\partial v}{\partial x} = 0, \quad \frac{\partial w}{\partial x} = 0, \quad N_1 = 0 \tag{3.4}$$

Acting in the similar way with the boundary conditions of the plate's $x = \text{const}$ free edge

$$\sigma_{11} + t_{11} = t_{11}^{(e)}, \quad \sigma_{12} + t_{12} = t_{12}^{(e)}, \quad \sigma_{13} + t_{13} = t_{13}^{(e)} \tag{3.5}$$

we can obtain the following conditions corresponding to the Kirchhoff plate theory conditions

$$T_1 = 0, \quad S + \frac{\mu h H_0^2}{2\pi}\frac{\partial v}{\partial x} = 0, \quad M_1 = 0, \quad \tilde{N}_1 + \frac{\mu h H_0^2}{2\pi}\frac{\partial w}{\partial x} = 0 \tag{3.6}$$

where $\tilde{N}_1$ is the generalized transverse shearing force

$$\tilde{N}_1 = N_1 + \frac{\partial H}{\partial y} \tag{3.7}$$

The conditions of the plate's $y = \text{const}$ edge are obtained in the similar way and have the following appearance:

for the hinge joint edge :

$$u = 0, \quad T_2 = 0, \quad w = 0, \quad M_2 = 0 \tag{3.8}$$

for the sliding contact

$$u = 0, \quad S = 0, \quad \frac{\partial w}{\partial y} = 0, \quad N_2 = 0 \tag{3.9}$$

for the free edge

$$T_2 + \frac{\mu h H_0^2}{2\pi}\frac{\partial v}{\partial y} = 0, \quad S = 0, \quad M_2 - \frac{\mu h^3 H_0^2}{6\pi}\frac{\partial^2 w}{\partial y^2} = 0, \quad \tilde{N}_2 = 0 \tag{3.10}$$

4. Now let us consider the localized bending vibrations of the plate [1] . Let us assume that the semi-infinite plate occupies the $0 \le x < \infty$, $-\infty < y < \infty$, $-h \le z \le h$ region. The localized waves are being propagated along the $x = 0$ edge. It is necessary to find the solution of (2.7) equation, which satisfies the boundary conditions on the $x = 0$ edge and the condition of damping

$$\lim_{x\to\infty} w = 0 \tag{4.1}$$

It is obvious that the solution of the (2.7) equation satisfying the (4.1) condition has the following appearance

$$w = \left(c_1 e^{-k p_1 x} + c_2 e^{-k p_2 x}\right)\exp i(\omega t - ky) \tag{4.2}$$

where

$$p_1 = \left(1 + \chi + \sqrt{\eta^2 + 2\chi + 2\chi^2}\right)^{1/2}, \quad p_2 = \left(1 + \chi - \sqrt{\eta^2 + 2\chi + \chi^2}\right)^{1/2}$$
$$\eta^2 = \frac{2\rho h\omega^2}{Dk^4}, \quad \chi = \frac{\mu h H_0^2}{4\pi D k^2} = \frac{3(1-\nu)}{4k^2h^2}\chi \tag{4.3}$$

The non-dimensional η^2 parameter in (4.3) defines the sought frequency of vibration, and according to the condition of damping (4.1), it should satisfy the following inequalities:

$$0 < \eta^2 < 1 \tag{4.4}$$

It is easy to check that the problem has no solution, satisfying the condition (4.4) , if the conditions of clamped edge or conditions for the the hinge joint (3.2) or the conditions of sliding contact (3.4) are set on the plate's $x = 0$ edge.

The boundary conditions of the free edge (3.6) for the bending vibrations of the plate, with taking into account the expressions for the moments and stresses, looks like

$$\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2} = 0, \quad \frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x \partial y^2} + \frac{\mu h H_0^2}{2\pi D}\frac{\partial w}{\partial x} = 0 \tag{4.5}$$

The substitution of the solution (4.2) into the (4.5) boundary conditions brings to a homogeneous equations system relative to the arbitrary c_1 c_2 constatns. The equalization to zero the determinant of the mentioned set defines the dispersion equation of the problem in the following way :

$$K(\eta) \equiv p_2(p_1^2 - \nu)[p_2^2 - (2 - \nu - 2\chi)] - p_1(p_2^2 - \nu)[p_1^2 - (2 - \nu - 2\chi)] = 0 \quad (4.6)$$

After some transformations the (4.6) equations result in

$$K(\eta) = (p_2 - p_1)K_1(\eta) \quad (4.7)$$

where

$$K_1(\eta) \equiv p_1^2 p_2^2 + 2(1 - \nu - \chi)p_1 p_2 - \nu^2 \quad (4.8)$$

Since the $p_1 \neq p_2$, we see that the frecuency of the vibrations can be obtained from the following equation:

$$K_1(\eta) = 0 \quad (4.9)$$

Noting that

$$K_1(0) = (1 - \nu)(3 + \nu) - 2\chi, \qquad K_1(1) = -\nu^2 < 0 \quad (4.10)$$

we shall obtain the sufficient condition of the (4.9) equation root existence

$$\chi < 0.5(3 + \nu)(1 - \nu), \quad \nu \neq 0 \quad (4.11)$$

This condition is also a necessary one, since it is easy to show that $K_1(\eta)$ function is monotonous in the $[0, 1]$ interval. The root of the (4.9) equation is being found in the following way

$$\eta^2 = 1 + 2(1 - \nu - \chi)\sqrt{(1 - \nu - \chi)^2 + \nu^2} - 2(1 - \nu - \chi)^2 - \nu^2 \quad (4.12)$$

From (4.11) and (4.3) it follows that satisfying the condition

$$\lambda \geq 2k^2h^2(1 + \nu/3) \quad (4.13)$$

the localized vibrations in the vicinity of the free edge of the plate are being eliminated. In particular, for the metal plate $(G \sim 10^{11} din/sm^2)$ and the relative wave-length $kh = 10^{-2}$ the induction of the intensity of the magnetic field $B_0 = \mu H_0$, necessary for eliminating the localized vibrations, is turned out an order of 1.5 tesla.

Now let us assume that the semi-infinite plate occupies the $-\infty < x < \infty$, $0 \leq y < \infty$, $-h \leq z \leq h$ region. We will consider the localized vibrations at the $y = 0$ free edge, taking into account the damping condition

$$\lim_{y \to \infty} w = 0 \quad (4.14)$$

In this case the solution of the equation (2.7), satisfying the (4.14) condition has the following appearance:

$$w = (c_1 e^{-k\Gamma_1 y} + c_2 e^{-k\Gamma_2 y})\exp i(\omega t - kx) \quad (4.15)$$

where

$$\Gamma_1 = \left(1+\sqrt{\eta^2-2\chi}\right)^{1/2}, \qquad \Gamma_2 = \left(1-\sqrt{\eta^2-2\chi}\right)^{1/2} \tag{4.16}$$

The designations for the η^2 and χ are the same as in (4.3). The condition of the existence of the localized vibrations is formulated in the following way:

$$2\chi < \eta^2 < 1+2\chi \tag{4.17}$$

According to (3.10), the boundary conditions of the free edge $y=0$ look like

$$\left(1+\frac{1-\nu}{2}\lambda\right)\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial x^2}=0, \quad \frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}=0 \tag{4.18}$$

From the requirement that the solution of (4.15) should satisfy the (4.18) boundary conditions we will obtain a dispersion equation, which can be brought to the following appearance after some definite transformations:

$$K_2(\eta)=\left(1+\frac{1-\nu}{2}\lambda\right)\Gamma_1^2\Gamma_2^2+2(1-\nu)\left(1+\frac{2-\nu}{4}\lambda\right)\Gamma_1\Gamma_2-\nu^2=0 \tag{4.19}$$

The calculation of the $K_2(\eta)$ function values at the ends of (4.17) interval results in

$$K_2(2\chi)=(1-\nu)\left(3+\nu+\frac{3-\nu}{2}\lambda\right)>0, \quad K_2(1+2\chi)=-\nu^2<0 \tag{4.20}$$

This implies for this problem the localized vibrations existence ($\nu \neq 0$) regardless of the magnitude of intensity of the magnetic field. We ought to notice that the influence of the magnetic field on the localized vibrations in the vicinity of $y=0$ free edge is essentially weak than it is in the vicinity of $x=0$ free edge. At the same time we should keep in mind that the intensity of the magnetic field should be also bounded by the $\lambda<1$ inequality.

References

1. Konenkov Y.K. On rayleigh type bending waves Akust Zh. 1960.V.6. №1. P.124-126 (in Russian).
2. Belubekyan M.V., Engibaryan I.A. Waves localized along the free edge of the plate with cubic symmetry // Proc of RAS. MTT. 1996. №6. P. 139-143 (in Russian).
3. Andronov I.V. Waves propagating along a narrow crack in an elastic plate. // Akust. Journ. 1999. V.45. №5. P. 444-449 (in Russian).

4. Gulgazaryan C.R., Gulgazaryan L.G. Vibration localized near the free end of semi-infinite moment free cylindrical shell.//Acoustic bulletin. 1999. V.2, №4. P. 42-48 (in Russian).
5. Kaliski S. Magnetoelastic vibration of a perfectly conducting plates and bars assuming the principle of plane sections. // Proc. of Vibr.Probl. 1962. V.3. №4. P. 225-234
6. Dunkin J.W., Eringen A.C. On the Propagation of Waves in electomagnetic elastic solid. // Jnt. J Eng. Sci 1963. V.1. P. 461-495.
7. Ambartsumian S.A., Bagdasaryan G.E., Belubekian M.V. On the tree-dimensional problem of the plate magnetoelastic vibration. // PMM. 1971. V.35. №2. P. 216-228.
8. Ambartsumian S.A., Bagdasaryan G.E., Belubekian M.V. The magnetoelasticity of thin sheels and plates.--M.: Nauka. 1977. 272p (in Russian).
9. Ambartsumian S.A., Belubekian M.V. On the approximate methods in the problems of the plate magnetoelastic vibration. In "Thermal stresses in the elements of constructions". Kiev. 1979. V.19. P. 3-6 (in Russian).
10. Ambartsumian S.A., Belubekian M.V. Vibration and stability of the current-caring elastic plates Yerevan. NAS of Armenia. 1992. 124 p. (in Russian).
11. Ambartsumian S.A., Bagdasaryan G.E. Conductive plates and shells in magnetic fields. M.: Phys-Math. lit. 1996. 288p. (in Russian).
12. Sindo Y., Ohnishi I., Toyama S. Dynamic singular moments in a perfectly conducting Mindlin plate with a through crack under a magnetic field.//ASME J. Appl. Mech. 2000. V.67. P. 503-510.

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