Известия НАН Армении, Математика, том 54, н. 6, 2019, стр. 19 – 27

A NEW FAMILY OF STARLIKE FUNCTIONS IN A CIRCULAR DOMAIN INVOLVING A q-DIFFERENTIAL OPERATOR

M. ARIF, Q. KHAN, J. SOKÓŁ

Abdul Wali Khan University Mardan, Pakistan
University of Rzeszów, Rzeszów, Poland
E-mails: *marifmaths@awkum.edu.pk*; *qaisermath84@gmail.com*; *jsokol@ur.edu.pl*

Abstract. The main purpose of the present paper is to investigate a number of useful properties such as sufficiency criteria, distortion bounds, coefficient estimates, radius of starlikness and radius of convexity for a new subclass of analytic functions, which are defined here by means of a newly defined q-linear differential operator.

MSC2010 numbers: 30C45, 30C50.
Keywords: analytic function; Janowski function; q-differential operator.

1. Introduction

Quantum calculus (q-calculus) is simply the study of classical calculus without the notion of limits. The study of q-calculus attracted the researcher due to its applications in various branches of mathematics and physics (see [6, 7]). Jackson [11, 12] was the first who gave some applications of the q-calculus and introduced the q-analogues of the derivative and integral. Later on, Aral and Gupta [5] – [7] defined the q-Baskakov-Durrmeyer operator by using the q-beta function, while Aral [4] and Anastassiu and Gal [8, 9] have discussed the q-generalizations of complex operators known as q-Picard and q-Gauss-Weierstrass singular integral operators. Recently, Kanas and Răducanu [13] defined a q-analogue of the Ruscheweyh differential operator by using the concept of convolution and then studied some of its properties. For more applications of this operator we refer the reader to the paper by Aldweby and Darus [3].

The aim of the present paper is to study some properties of a new family of starlike functions associated with a circular domain involving a q-differential operator.

Let $\mathfrak{A}$ denote the family of all normalized analytic functions f in the open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ obeying the normalization:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \; z \in \mathbb{D}. \tag{1.1}$$

Also, let $\mathcal{S}^*(\alpha)$ and $\mathcal{K}(\alpha)$ denote the well-known families of starlike and convex functions of order α $(0 \leq \alpha < 1)$, respectively. For two functions f and g that are analytic in $\mathbb{D}$ and have the form (1.1), we define their convolution by

$$f(z) * g(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k, \; z \in \mathbb{D}.$$

For $0 < q < 1$, the q-derivative of a function f is defined by

$$\partial_q f(z) = \frac{f(qz) - f(z)}{z(q-1)}, \; z \neq 0. \tag{1.2}$$

If $f(0) = 0$, then $\partial_q f(z)$ is well defined also at $z = 0$. It can easily be seen that for $n \in \mathbb{N} = \{1, 2, 3, \ldots\}$ and $z \in \mathbb{D}$ we have

$$\partial_q \left\{ \sum_{n=1}^{\infty} a_n z^n \right\} = \sum_{n=1}^{\infty} [n, q]\, a_n z^{n-1}, \tag{1.3}$$

where

$$[n, q] = \frac{1 - q^n}{1 - q} = 1 + \sum_{l=1}^{n-1} q^l, \quad [0, q] = 0.$$

For any non-negative integer n the q-number shift factorial is defined by

$$[n, q]! = \begin{cases} 1, \; n = 0, \\ [1, q]\,[2, q]\,[3, q] \cdots [n, q]\,, \; n \in \mathbb{N}. \end{cases}$$

Also, the q-generalized Pochhammer symbol for $x > 0$ is given by

$$[x, q]_n = \begin{cases} 1, \; n = 0, \\ [x, q][x + 1, q] \cdots [x + n - 1, q], \; n \in \mathbb{N}, \end{cases}$$

and for $x > 0$, let the q-gamma function be defined as

$$\Gamma_q(x + 1) = [x, q]\, \Gamma_q(t) \text{ and } \Gamma_q(1) = 1.$$

We now define the function:

$$\Phi(q, \mu + 1; z) = z + \sum_{n=2}^{\infty} \wedge_n \, z^n, \; \mu > -1, \; z \in \mathbb{D}, \tag{1.4}$$

with

$$\wedge_n = \frac{[\mu + 1, q]_{n+1}}{[n + 1, q]!}.$$

Using the function $\Phi(q, \mu; z)$ and the definition of q-derivative, Kanas and Răducanu [13] defined the differential operator $\mathcal{L}_q^\mu : \mathfrak{A} \to \mathfrak{A}$ by

$$\mathcal{L}_q^\mu f(z) = \Phi(q, \mu; z) * f(z) = z + \sum_{n=2}^{\infty} \wedge_n \, a_n z^n, \; \mu > -1, \; z \in \mathbb{D}. \tag{1.5}$$

We note that $\mathcal{L}_q^0 f(z) = f(z)$, $\mathcal{L}_q^1 f(z) = z\partial_q f(z)$, and

$$\mathcal{L}_q^m f(z) = \frac{z\partial_q^m \left(z^{m-1} f(z)\right)}{[m, q]!}, \quad m \in \mathbb{N}.$$

Note that when $q \to 1^-$, the q-differential operator defined in (1.5) reduces to the familiar differential operator introduced in [17].

From (1.5) , one can easily obtain the following identity

$$[\mu+1,q]\mathcal{L}_q^{\mu+1}f(z)=[\mu,q]\mathcal{L}_q^{\mu}f(z)+q^{\mu}z\partial_q\left(\mathcal{L}_q^{\mu}f(z)\right). \tag{1.6}$$

For more details on the q-analogue of differential operators we refer to the papers [1, 2, 15].

Motivated from the works [10, 18], we now define a subfamily $\mathcal{S}_q^*(\mu,A,B)$ of $\mathfrak{A}$ by using the operator $\mathcal{L}_q^{\mu}$ as follows.

Definition 1.1. Let $-1\le B<A\le 1$, $\mu>-1$ and $0<q<1$. We say that a function $f\in\mathfrak{A}$ is of the class $\mathcal{S}_q^*(\mu,A,B)$ if it satisfies the condition:

$$\frac{z\partial_q\mathcal{L}_q^{\mu}f(z)}{\mathcal{L}_q^{\mu}f(z)}\prec\frac{1+Az}{1+Bz}, \tag{1.7}$$

where the symbol "$\prec$"stands for the familiar subordination.

Equivalently, a function $f\in\mathfrak{A}$ is of the class $\mathcal{S}_q^*(\mu,A,B)$ if and only if

$$\left|\frac{\frac{z\partial_q\mathcal{L}_q^{\mu}f(z)}{\mathcal{L}_q^{\mu}f(z)}-1}{A-B\frac{z\partial_q\mathcal{L}_q^{\mu}f(z)}{\mathcal{L}_q^{\mu}f(z)}}\right|<1,\ z\in\mathbb{D}. \tag{1.8}$$

Lemma 1.2 ([16])**.** *Let a function $h(z)$ be analytic in $\mathbb{D}$ with series representation:*

$$h(z)=1+\sum_{n=1}^{\infty}d_nz^n,$$

and let a function $k(z)$ be analytic and convex univalent in $\mathbb{D}$ with series representation:

$$k(z)=1+\sum_{n=1}^{\infty}k_nz^n.$$

If $h(z)\prec k(z)$, then $|d_n|\le|k_1|$, for $n\in\mathbb{N}:=\{1,2,\ldots\}$.

Lemma 1.3 ([14])**.** *Let $-1\le B_2\le B_1<A_1\le A_2\le 1$. Then*

$$\frac{1+A_1z}{1+B_1z}\prec\frac{1+A_2z}{1+B_2z}.$$

2. The main results

Theorem 2.1. *Let $f\in\mathfrak{A}$ be a function of the form* (1.1)*. Then $f\in\mathcal{S}_q^*(\mu,A,B)$ if and only if the following inequality holds:*

$$\sum_{n=1}^{n}\wedge_n\left([n,q](1-B)-(1-A)\right)|a_n|\le(A-B). \tag{2.1}$$

Proof. We first assume that the inequality (2.1) holds. To show that $f\in\mathcal{S}_q^*(\mu,A,B)$, we only need to prove the inequality (1.8). To this end, we first use

(1.5), and then (1.2) and (1.3) , to obtain

$$\begin{aligned}
\left|\frac{\frac{z\partial_q\mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)}-1}{A-B\frac{z\partial_q\mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)}}\right| &= \left|\frac{z\partial_q\mathcal{L}_q^\mu f(z)-\mathcal{L}_q^\mu f(z)}{A\mathcal{L}_q^\mu f(z)-Bz\partial_q\mathcal{L}_q^\mu f(z)}\right| \\
&= \left|\frac{z+\sum_{n=2}^\infty \wedge_n a_n [n,q] z^n-\left(z+\sum_{n=2}^\infty \wedge_n a_n z^n\right)}{A\left(z+\sum_{n=2}^\infty \wedge_n a_n z^n\right)-B\left(z+\sum_{n=2}^\infty \wedge_n a_n [n,q] z^n\right)}\right| \\
&= \left|\frac{\sum_{n=2}^\infty \wedge_n a_n\left([n,q]-1\right) z^n}{(A-B)z+\sum_{n=2}^\infty \wedge_n a_n\left(A-B[n,q]\right) z^n}\right| \\
&\le \frac{\sum_{n=2}^\infty \wedge_n |a_n|\left([n,q]-1\right)|z|^n}{(A-B)|z|-\sum_{n=2}^\infty \wedge_n |a_n|\left(A-B[n,q]\right)|z|^n} \\
&\le \frac{\sum_{n=2}^\infty \wedge_n |a_n|\left([n,q]-1\right)}{(A-B)-\sum_{n=2}^\infty \wedge_n |a_n|\left(A-B[n,q]\right)}<1.
\end{aligned}$$

This completes the direct part of the proof.

Conversely, let $f\in\mathcal{S}_q^*(\mu,A,B)$ be of the form (1.1) . Then from (1.8) along with (1.5), we have for $z\in\mathbb{D}$,

$$\left|\frac{\frac{z\partial_q\mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)}-1}{A-B\frac{z\partial_q\mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)}}\right| = \left|\frac{\sum_{n=2}^\infty \wedge_n a_n\left([n,q]-1\right) z^n}{(A-B)z-\sum_{n=2}^\infty \wedge_n a_n\left(A-B[n,q]\right) z^n}\right|<1.$$

Since $|\mathfrak{Re} z|\le|z|$, we have

$$\mathfrak{Re}\left\{\frac{\sum_{n=2}^\infty \wedge_n a_n\left([n,q]-1\right) z^n}{(A-B)z-\sum_{n=2}^\infty \wedge_n a_n\left(A-B[n,q]\right) z^n}\right\}<1. \tag{2.2}$$

Now choose the values of z on the real axis so that $\frac{z\partial_q\mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)}$ is real. Upon clearing the denominator in (2.2) and letting $z\to 1^-$ through real values, we obtain (2.1). □

Theorem 2.2. *Let a function $f\in\mathcal{S}_q^*(\mu,A,B)$ be of the form (1.1). Then the following inequalities hold:*

$$\begin{aligned}
|a_2| &\le \frac{A-B}{([2,q]-1)\wedge_2}, \\
|a_3| &\le \frac{A-B}{([3,q]-1)\wedge_3}\left(1+\frac{A-B}{([2,q]-1)}\right), \\
|a_4| &\le \frac{A-B}{([4,q]-1)\wedge_4}\left(1+\frac{A-B}{([2,q]-1)}+\frac{A-B}{([3,q]-1)}\right).
\end{aligned}$$

Proof. If $f\in\mathcal{S}_q^*(\mu,A,B)$, then we have

$$\frac{z\partial_q\mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)}\prec\frac{1+Az}{1+Bz}.$$

Let

$$h(z)=\frac{z\partial_q\mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)}, \tag{2.3}$$

and be of the form:

$$h(z)=1+\sum_{n=1}^{\infty}d_nz^n.$$

Since

$$h(z)\prec\frac{1+Az}{1+Bz}=1+(A-B)z+\cdots,$$

then by Lemma 1.2 we get

$$|d_n|\leq A-B. \tag{2.4}$$

Now substituting the series forms of $h(z)$ and $f(z)$ into (2.3), and then simplifying and comparing the coefficients of z^n on both sides, we get

$$\wedge_n\left([n,q]-1\right)a_n \quad = \quad \wedge_{n-1}a_{n-1}d_1 \quad + \quad \wedge_{n-2}a_{n-2}d_2 \quad +\cdots+ \quad \wedge_1a_1d_{n-1}.$$

Taking absolute value on both sides and then using (2.4), we obtain

$$\wedge_n\left([n,q]-1\right)|a_n|\leq(A-B)\sum_{k=1}^{n-1}\wedge_k|a_k|,$$

implying that

$$|a_n|\leq\frac{(A-B)}{([n,q]-1)\wedge_n}\sum_{k=1}^{n-1}\wedge_k|a_k|.$$

Now taking $n=2,3,4$ and using that $|a_1|=1$ we get the desired result. □

Theorem 2.3. *Let a function* $f\in\mathcal{S}_q^*(\mu,A,B)$ *be of the form* (1.1). *Then for* $|z|=r$ *the following inequalities hold:*

$$r-\frac{(A-B)[2,q]!}{[\mu+1,q]_2([2,q](1-B)-(1-A))} \quad \leq \quad |f(z)| \quad \leq \quad r+\frac{(A-B)[2,q]!}{[\mu+1,q]_2([2,q](1-B)-(1-A))}.$$

Proof. We have

$$|f(z)| \quad = \quad \left|z+\sum_{n=2}^{\infty}a_n\ z^n\right|\leq|z|+\sum_{n=2}^{\infty}|a_n|\ \ |z|^n=r+\sum_{n=2}^{\infty}|a_n|\ \ r^n.$$

Since $|z|=r<1$, we have $r^n<r<1$ and

$$|f(z)|\leq r+\sum_{n=2}^{\infty}|a_n|. \tag{2.5}$$

Similarly, we get

$$|f(z)|\geq r-\sum_{n=2}^{\infty}|a_n|. \tag{2.6}$$

Since $f\in\mathcal{S}_q^*(\mu,A,B)$, by (2.1) we get

$$\sum_{n=2}^{\infty}\frac{[\mu+1,q]_{n+1}}{[n+1,q]!}\left([n,q](1-B)-(1-A)\right)|a_n| \quad \leq \quad A-B.$$

On the other hand, we have

$$\tfrac{[\mu+1,q]_2}{[2,q]!}\left([n,q]\left(1-B\right)-\left(1-A\right)\right)\sum_{n=2}^{\infty}|a_n|$$
$$\leq \sum_{n=2}^{\infty}\tfrac{[\mu+1,q]_{n+1}}{[n+1,q]!}\left([n,q]\left(1-B\right)-\left(1-A\right)\right)|a_n|.$$

Hence

$$\tfrac{[\mu+1,q]_2}{[2,q]!}\left([n,q]\left(1-B\right)-\left(1-A\right)\right)\sum_{n=2}^{\infty}|a_n| \quad\leq\quad A-B,$$

implying that

$$\sum_{n=2}^{\infty}|a_n| \quad\leq\quad \tfrac{(A-B)[2,q]!}{([n,q](1-B)-(1-A))[\mu+1,q]_2}.$$

Finally, combining this inequality with (2.5) and (2.6) , we get the required result. □

Theorem 2.4. *Let a function $f\in\mathcal{S}_q^*(\mu,A,B)$ be of the form (1.1). Then for $|z|=r$ the following inequalities hold:*

$$\frac{(A-B)\,[2,q]!}{\left([n,q]\left(1-B\right)+\left(1-A\right)\right)}r\leq\left|\partial_q^m f(z)\right|\leq\frac{(A-B)\,[2,q]!}{\left([n,q]\left(1-B\right)-\left(1-A\right)\right)}r.$$

Proof. By the virtue of (1.2) and (1.3) , we can write

$$\partial_q^m f(z)=\sum_{n=2}^{\infty}[n,q]_m a_n z^{n-m}.$$

Since $|z|=r<1$, we have $r^{n-m}\leq r$ for $m<n$, and hence

$$\left|\partial_q^m f(z)\right|\leq r\sum_{n=2}^{\infty}[n,q]_m\,|a_n|\,. \tag{2.7}$$

Similarly, we get

$$\left|\partial_q^m f(z)\right|\geq -r\sum_{n=2}^{\infty}[n,q]_m\,|a_n|\,. \tag{2.8}$$

Now, using (2.1) and the following inequality

$$\tfrac{([2,q](1-B)-(1-A))}{[2,q]!}\sum_{n=2}^{\infty}[\mu+1,q]_{n+1}\,|a_n|$$
$$\leq\sum_{n=2}^{\infty}\tfrac{[\mu+1,q]_{n+1}}{[n+1,q]!}\left([2,q]\left(1-B\right)-\left(1-A\right)\right)|a_n|\,,$$

we obtain

$$\sum_{n=2}^{\infty}[\mu+1,q]_{n+1}\,|a_n|\leq\frac{(A-B)\,[2,q]!}{\left([2,q]\left(1-B\right)-\left(1-A\right)\right)}.$$

On the other hand, we have

$$\sum_{n=2}^{\infty}[n,q]_m\,|a_n|\leq\sum_{n=1}^{\infty}[\mu+1,q]_{n+1}\,|a_n|\,,$$

implying that

$$\sum_{n=2}^{\infty}[n,q]_m\,|a_n|\leq\frac{(A-B)\,[2,q]!}{\left([2,q]\left(1-B\right)-\left(1-A\right)\right)}.$$

Finally, combining this inequality with (2.7) and (2.8), we obtain the required result. □

Theorem 2.5. *Let* $f \in \mathcal{S}_q^*(\mu, A, B)$. *Then* $f \in \mathcal{S}^*(\alpha)$ *for* $|z| < r_1$, *where*

$$r_1 = \left(\frac{(1-\alpha)[\mu+1,q]_n ([n,q](1-B)-(1-A))}{(n-\alpha)(A-B)[n,q]!} \right)^{\frac{1}{n-1}}.$$

Proof. Let $f \in \mathcal{S}_q^*(\mu, A, B)$. To prove that $f \in \mathcal{S}^*(\alpha)$, we only need to show that

$$\left| \frac{\frac{zf'(z)}{f(z)} - 1}{\frac{zf'(z)}{f(z)} + 1 - 2\alpha} \right| < 1.$$

Using (1.1) along with some simple computation, we get

$$\sum_{n=2}^{\infty} \left(\frac{n-\alpha}{1-\alpha} \right) |a_n| |z|^{n-1} < 1. \tag{2.9}$$

Since $f \in \mathcal{S}_q^*(\mu, A, B)$, from (2.1), we can easily obtain that

$$\sum_{n=2}^{\infty} \frac{[\mu+1,q]_n}{[n,q]!} \left(\frac{[n,q](1-B)-(1-A)}{A-B} \right) |a_n| < 1.$$

Now the inequality (2.9) will be satisfied if the following holds:

$$\sum_{n=2}^{\infty} \left(\frac{n-\alpha}{1-\alpha} \right) |a_n| |z|^{n-1} < \sum_{n=2}^{\infty} \frac{[\mu+1,q]_n}{[n,q]!} \left(\frac{[n,q](1-B)-(1-A)}{A-B} \right) |a_n|,$$

which implies that

$$|z|^{n-1} < \frac{(1-\alpha)([n,q](1-B)-(1-\alpha))[\mu+1,q]_n}{(n-\alpha)(A-B)[n,q]!}.$$

Therefore

$$|z| < \left(\frac{(1-\alpha)([n,q](1-B)-(1-A))[\mu+1,q]_n}{(n-\alpha)(A-B)[n,q]!} \right)^{\frac{1}{n-1}} = r_1,$$

and the result follows. □

Theorem 2.6. *Let* $f \in \mathcal{S}_q^*(\mu, A, B)$. *Then* $f \in \mathcal{C}(\alpha)$ *for* $|z| < r_2$, *where*

$$r_2 = \left(\frac{(1-\alpha)([n,q](1-B)-(1-A))[\mu+1,q]_n}{n(n-\alpha)(A-B)[n,q]!} \right)^{\frac{1}{n-1}}.$$

Proof. We know that $f \in \mathcal{C}(\alpha)$ if and only if

$$\left| \frac{\frac{(zf'(z))'}{f'(z)} - 1}{\frac{(zf'(z))'}{f'(z)} + 1 - 2\alpha} \right| < 1.$$

Using (1.1) and some simplification, we get

$$\sum_{n=2}^{\infty} \left(\frac{n(n-\alpha)}{1-\alpha} \right) |a_n| |z|^{n-1} < 1. \tag{2.10}$$

Next, from (2.1) we can easily obtain

$$\sum_{n=2}^{\infty} \frac{[\mu+1,q]_n}{[n,q]!} \left(\frac{([n,q](1-B)-(1-A))}{A-B}\right) |a_n| \quad < \quad 1.$$

For inequality (2.10) to be true it will be enough to have

$$\sum_{n=2}^{\infty} \left(\frac{n(n-\alpha)}{1-\alpha}\right) |a_n| |z|^{n-1} < \sum_{n=2}^{\infty} \frac{[\mu+1,q]_n}{[n,q]!} \left(\frac{([n,q](1-B)-(1-A))}{A-B}\right) |a_n| .$$

This gives

$$|z|^{n-1} < \left(\frac{(1-\alpha)([n,q](1-B)-(1-A))[\mu+1,q]_n}{n(n-\alpha)(A-B)[n,q]!}\right),$$

and hence

$$|z| < \left(\frac{(1-\alpha)([n,q](1-B)-(1-A))[\mu+1,q]_n}{n(n-\alpha)(A-B)[n,q]!}\right)^{\frac{1}{n-1}} = r_2,$$

and the result follows. □

Theorem 2.7. *Let* $-1 \le B_2 \le B_1 < A_1 \le A_2 \le 1$, *and let* $\mathcal{L}_q^\mu f(z) \neq 0$ *in* $\mathbb{D}$ *and satisfy*

$$\frac{1}{q^\mu}\left(\frac{[\mu+1,q]\,\mathcal{L}_q^{\mu+1} f(z)}{\mathcal{L}_q^\mu f(z)} - [\mu,q]\right) \prec \frac{1+A_1 z}{1+B_1 z}. \tag{2.11}$$

Then $f \in \mathcal{S}_q^*(\mu, A_2, B_2)$.

Proof. Since $\mathcal{L}_q^\mu f(z) \neq 0$ in $\mathbb{D}$, we can define the function $p(z)$ by

$$\frac{z\partial_q \mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)} = p(z) \quad (z \in \mathbb{D}).$$

In view of identity (1.6), we easily obtain

$$\frac{1}{q^\mu}\left(\frac{[\mu+1,q]\,\mathcal{L}_q^{\mu+1} f(z)}{\mathcal{L}_q^\mu f(z)} - [\mu,q]\right) = p(z).$$

Therefore, using (2.11), we get

$$\frac{z\partial_q \mathcal{L}_q^\mu f(z)}{\mathcal{L}_q^\mu f(z)} = p(z) \prec \frac{1+A_1 z}{1+B_1 z}.$$

Applying Lemma 1.3, we conclude that $f \in \mathcal{S}_q^*(\mu, A_2, B_2)$. □

Список литературы

[1] I. Aldawish and M. Darus, “Starlikness of q-differential operator involving quantum calculus”, Korean J. Math., **22**, no. 4, 699 – 709 (2014).

[2] H. Aldweby and M. Darus, “A subclass of harmonic univalent functions associated with q-analogue of Dziok-Srivastava operator”, ISRN Math. Anal., **2013**, Article ID 382312, 6 pages.

[3] H. Aldweby and M. Darus, “Some subordination results on q -analogue of Ruscheweyh differential operator”, Abstr. Appl. Anal., **2014**, Article ID 958563, 6 pages.

[4] A. Aral, “On the generalized Picard and Gauss Weierstrass singular integrals”, J. Compu. Anal. Appl., **8**, no. 3, 249 – 261 (2006).

[5] A. Aral and V. Gupta, “Generalized q-Baskakov operators”, Math. Slovaca, **61**, no. 4, 619 – 634.

[6] A. Aral and V. Gupta, "On the Durrmeyer type modification of the q-Baskakov type operators", Non-linear Anal. Theory, Methods and Appl., **72**, nn. 3-4, 1171 – 1180 (2010).

[7] A. Aral and V. Gupta, "On q-Baskakov type operators", Demonstratio Mathematica, **42**, no. 1, 109 – 122 (2009).

[8] G. A. Anastassiu and S. G. Gal, "Geometric and approximation properties of generalized singular integrals", J. Korean Math. Soci., **23**, no. 2, 425 – 443 (2006).

[9] G. A. Anastassiu and S. G. Gal, "Geometric and approximation properties of some singular integrals in the unit disk", J. Ineq. Appl., **2006**, Article ID 17231, 19 pages.

[10] A. Huda and M. Darus, "Integral operator defined by q-analogue of Liu-Srivastava operator", Studia Universitatis Babes-Bolyai, Series Mathematica, **58**, no. 4, 529 – 537 (2013).

[11] F. H. Jackson, "On q-definite integrals", The Quarterly J. Pure Appl. Math., **41**, 193 – 203 (1910).

[12] F. H. Jackson, "On q-functions and a certain difference operator", Trans. Royal Soc. Edinburgh, **46**, no. 2, 253 – 281 (1909).

[13] S. Kanas and D. Răducanu, "Some class of analytic functions related to conic domains", Math. Slovaca, **64**, no. 5, 1183 – 1196 (2014). .

[14] M. S. Liu, "On a subclass of p-valent close to convex functions of type α and order β" [in Chinese], J. Math. Study, **30**, no. 1, 102 – 104 (1997).

[15] A. Mohammed and M. Darus, "A generalized operator involving the q-hypergeometric function", Mat. Vesnik, **65**, no. 4, 454 – 465 (2013).

[16] W. Rogosinski, "On the coefficients of subordinate functions", Proc. London Math. Soc., **48**, no. 2, 48 – 82 (1943).

[17] St. Ruscheweyh, "New criteria for univalent functions", Proc. Amer. Math. Soc., **49**, 109 – 115 (1975).

[18] T. M. Seoudy and M. K. Aouf, "Coefficient estimates of new classes of q-starlike and q-convex functions of complex order", J. Math. Inequ., **10**, no. 1, 135 – 145 (2016).

Поступила 28 июня 2017

После доработки 5 октября 2017

Принята к публикации 12 октября 2017