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SIMULATION OF A RANDOM FUZZY QUEUING SYSTEM WITH MULTIPLE SERVERS

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Abstract. The paper considers a queuing system that has k servers and its interarrival times and service times are random fuzzy variables. We obtain a new theorem concerning the average chance of the event "r servers ($r \leq k$) are busy at time t", provided that all the servers work independently. We simulate the average chance using fuzzy simulation method and obtain some results on the number of servers that are busy. Some examples to illustrate the simulation procedure are also presented.

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Keywords: Multi-server queuing system; busy time; fuzzy interarrival times; average chance.

1. Introduction

Queuing systems constitute a central tool in modeling and performance of telecommunication and computer systems. In the fuzzy case, it is assumed that the interarrival times and the service times are random fuzzy variables. Pardoa and Fuenteb [1] proposed the analysis, development and design of a fuzzy queuing model with a finite input source in which the arrival pattern as well as the service pattern follow an exponential distribution with an uncertain parameter. Wanga, Liub and Watada [2] studied a fuzzy random renewal process in which the interarrival times are assumed to be independent and identically distributed fuzzy random variables, and two case studies of queuing systems are provided to illustrate the application of the fuzzy random elementary renewal theorem. Wu [3] has proposed the fuzzy arrival rate and fuzzy service rate in a queuing system. The nonhomogeneous Poisson process with fuzzy intensity function is taken as the arrival process for this queuing system. Computational procedures for performing simulation in the α-level sense and for obtaining the α-level closed intervals of the system performance measure are also proposed to tackle this kind of model. Chen [4] has proposed a procedure for constructing the membership functions of the performance measures in finite-capacity queuing systems with the arrival rate and service rate being fuzzy numbers. Kreimer [5] studied a real-time multi-server system with homogeneous servers (such as unmanned air vehicles or machine controllers) and several nonidentical channels (such as surveillance regions or assembly lines), working under maximum load regime. Zhao, Li and Huang [6] developed a queue-based interval-fuzzy electric-power system (QIF-ESP) model through coupling fuzzy queue (FQ) theory with interval-parameter programming (IPP). Yang and Chang [7] investigated the F-policy queue using fuzzy parameters, in which the arrival rate, the service rate, and the start-up rate are all fuzzy numbers. The F-policy deals with the control of arrivals in a queuing system, in which the server requires a start-up time before allowing customers to enter.

In this paper, we simulate the average chance of the event "all the k servers are busy at time t"and the queuing system has k servers. In other words, we estimate the average chance of the event "r servers ($r \leq k$) are busy at time t when all the servers work independently and the interarrival times and the service times are random fuzzy variables. We obtain some results about the relationship between the number of servers and busy times and idle times.

The paper is structured as follows. In Section 2, we discuss the concepts and essential properties of fuzzy set theory, fuzzy variables, random fuzzy variables, the average chance, etc. In Section 3, we illustrate the random fuzzy queuing system with multiple servers and estimate the average chance of the event "r servers ($r \leq k$) are busy at time t". In Section 4, we consider the fuzzy simulation method. In Section 5 we provide some numerical examples.

2. Definitions and Preliminaries

Credibility theory, introduced by Liu (see [8]), is a branch of mathematics for studying the behavior of fuzzy phenomena. In this section, we introduce the basic notions of credibilty theory, such us credibility measure, credibility space, fuzzy variable, membership function, credibility distribution, expected value, random fuzzy variable and its expected value, independence and identical distribution.

Let Θ be a nonempty set, and P be the power set of Θ, that is, the largest σ-algebra over Θ. Each element of P is called an *event*. In order to give an axiomatic definition of credibility, it is necessary to assign to each event A a number $Cr\{A\}$ which indicates the credibility that A will occur.

Definition 2.1. *(Liu and Liu [9]). A set function Cr defined on Θ is called a credibility measure if it satisfies the following axioms:*
Axiom 1. *(Normality): $Cr\{\Theta\} = 1$.*
Axiom 2. *(Monotonicity): $Cr\{A\} \leq Cr\{B\}$ for $A \subset B$.*
Axiom 3. *(Self-Duality): $Cr\{A\} + Cr\{A^c\} = 1$ for any event A.*
Axiom 4. *(Maximality): $C\{\cup_i A_i\} = \sup_i Cr\{A_i\}$ for any events $\{A_i\}$ with $\sup_i Cr\{A_i\} <$ 0.5.*

Then the triplet (Θ, P, Cr) is called a credibility space. The product credibility measure can be defined in multiple ways. We accept the following axiom.

Axiom 5 *(Product Credibility Axiom). Let Θ_k be nonempty sets on which Cr_k are credibility measures for $k = 1, 2, \ldots, n$, respectively, and let $\Theta = \Theta_1 \times \Theta_2 \times \ldots \times \Theta_n$. Then*

$$Cr\{(\theta_1, \theta_2, \ldots, \theta_n)\} = Cr_1\{\theta_1\} \wedge Cr_2\{\theta_2\} \wedge \ldots \wedge Cr_n\{\theta_n\} \tag{2.1}$$

for each $(\theta_1, \theta_2, \ldots, \theta_n) \in \Theta$.

Let (θ_k, P_k, Cr_k), $k = 1, 2, \ldots, n$, be credibility spaces, $\Theta = \Theta_1 \times \Theta_2 \times \ldots \times \Theta_n$ and $Cr = Cr_1 \wedge Cr_2 \wedge \ldots \wedge Cr_n$. Then the triplet (Θ, P, Cr) is called the product credibility space of (θ_k, P_k, Cr_k), $k = 1, 2, \ldots, n$.

Definition 2.2. *A fuzzy variable is defined to be any real-valued measurable function defined on a credibility space (Θ, P, Cr).*

Definition 2.3. *Let ξ be a fuzzy variable defined on the credibility space (Θ, P, Cr). Then the membership function μ of ξ is defined by*

$$\mu(x) = (2Cr\{\xi = x\}) \wedge 1, \; x \in \Re. \tag{2.2}$$

Definition 2.4. *Let ξ be a fuzzy variable defined on the credibility space $(\Theta, P(\Theta), Cr)$ and let $\alpha \in (0, 1]$. Then*

$$\xi'_\alpha = \inf\{r | \mu_\xi(r) \geq \alpha\}, \quad \xi''_\alpha = \sup\{r | \mu_\xi(r) \geq \alpha\} \tag{2.3}$$

are called α-pessimistic value and α-optimistic value of ξ, respectively.

Definition 2.5. *Let ξ be a fuzzy variable defined on the credibility space $(\Theta, P(\Theta), Cr)$. Then the expected value of ξ is defined by*

$$E[\xi] = \int_0^\infty Cr\{\xi \geq r\}dr - \int_{-\infty}^0 Cr\{\xi \leq r\}dr, \tag{2.4}$$

provided that at least one of the two integrals in (2.4) is finite (see [10]).

Proposition 2.1. *Let ξ be a fuzzy variable defined on the credibility space $(\Theta, P(\Theta), Cr)$. Then we have*

$$E[\xi] = \frac{1}{2}\int_0^1 [\xi'_\alpha + \xi''_\alpha]d\alpha. \tag{2.5}$$

Proof. Let ξ be normalized, that is, there exists a real number r_0 such that $\mu_\xi(r_0) = 1$. If $r_0 > 0$, then in view of (2.1), we have

$$E[\xi] = \frac{1}{2}[r_0 + \int_{r_0}^{+\infty} Cr(\xi \geq r)dr + r_0 - \int_{-\infty}^{r_0} Cr(\xi \leq r)dr] = \frac{1}{2}\int_0^1 (\xi'_\alpha + \xi''_\alpha)d\alpha,$$

implying (2.5). The case $r_0 \leq 0$ can be treated similarly. □

Definition 2.6. *The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_m$ are called independent if*

$$Cr\{\cap_{i=1}^m \{\xi_i \in B_i\}\} = \min_{1 \leq i \leq m} Cr\{\xi_i \in B_i\} \tag{2.6}$$

for any sets $B_1, B_2, \ldots, B_m \in \Re$.

Definition 2.7. *A random fuzzy variable is defined to be any function from the credibility space (Θ, P, Cr) to the set of random variables.*

Definition 2.8. *The expected value of a random fuzzy variable ξ is defined by*

$$E[\xi] = \int_0^\infty Cr\{\theta \in \Theta | E[\xi(\theta)] \geq r\}dr - \int_{-\infty}^0 Cr\{\theta \in \Theta | E[\xi(\theta)] \leq r\}dr. \tag{2.7}$$

Proposition 2.2. *Let ξ be a random fuzzy variable defined on the credibility space (Θ, P, Cr). Then for any $\theta \in \Theta$ the expected value $E[\xi(\theta)]$ is a fuzzy variable, provided that $E[\xi(\theta)]$ is finite for a fixed $\theta \in \Theta$.*

Definition 2.9. *The random fuzzy variables ξ and η are said to be identically distributed if*

$$\sup_{Cr\{A\} \geq \alpha} \inf_{\theta \in A} \{Pr\{\xi(\theta) \in B\}\} = \sup_{Cr\{A\} \geq \alpha} \inf_{\theta \in A} \{Pr\{\eta(\theta) \in B\}\} \tag{2.8}$$

for any $\alpha \in (0, 1]$ and any Borel set B of real numbers.

Definition 2.10. *The random fuzzy variables ξ_i, $i = 1, \dots, n$, are said to be independent if*

(1) $\xi_i(\theta)$, $i = 1, \dots, n$, are independent random variables for each $\theta \in \Theta$.
(2) $E[\xi_i(\cdot)]$, $i = 1, \dots, n$, are independent fuzzy variables.

Definition 2.11. *Let ξ be a random fuzzy variable defined on the credibility space $(\Theta, P(\Theta), Cr)$. Then the average chance of the random fuzzy event $\xi \leq 0$ is defined as*

$$Ch\{\xi \leq 0\} = \int_0^1 Cr\{\theta \in \Theta | Pr\{\xi(\theta) \leq 0\} \geq p\} dp. \tag{2.9}$$

3. Random fuzzy queuing systems with k servers

In this section we study a model of queuing system with k servers, denoted by $RF/RF/k/FCFS/\infty/\infty$, where RF/RF means that the interarrival times and the service times are random fuzzy variables, $FCFS$ means that the queue discipline is "first come, first served and the size of source population is infinite. We assume that the interarrival times of customers arriving at the server are independent and identically distributed random fuzzy variables. $\xi_i \sim EXP(\lambda_i)$, where λ_i are fuzzy variables defined on the credibility space $(\Theta_i, P(\Theta_i), Cr_i)$, $i = 1, 2, \dots$, and the service times are independent and identically distributed random fuzzy variables, $\eta_i \sim EXP(\mu_i)$, where μ_i are fuzzy variables defined on the credibility space $(\Gamma_i, P(\Gamma_i), Cr_i')$, $i = 1, 2, \dots$ and ξ_i and η_i are independent.

For the model $RF/RF/k/FCFS/\infty/\infty$ we describe the limit (as $t \to \infty$) of the average chance of the event "the random fuzzy queuing system is busy at time t when the queuing system has k servers. The case of different number of servers is also discussed. Notice that in the special case where the model involves only one server ($k = 1$). this problem has been considered in [11].

Define $P(t) = Pr\{all\ of\ k\ servers\ are\ busy\ at\ time\ t\}$, and $P_i(t) = Pr\{the\ ith\ server\ is\ busy\ at\ time\ t\}$, and observe that $P(t)$ and $P_i(t)$ are fuzzy variables, P'_{α_0} and P''_{α_0} are the α_0-pessimistic values and the α_0-optimistic values of $P(t)$, respectively, and $E[\frac{\lambda}{\mu}] < 1$.

Lemma 3.1. *([11]). Assume that in a random fuzzy queuing system $RF/RF/k/FCFS/\infty/\infty$, the fuzzy variable λ has the same α_0-pessimistic values and the α_0-optimistic values λ_i, and the fuzzy variable μ has the same α_0-pessimistic values and the α_0-optimistic values μ_i, and are continuous at the point $\alpha_0 \in [0, 1]$. Also, let the k servers work* *independently, then we have*

$$\lim_{t\to\infty} P'_{i\alpha}(t) = \frac{\lambda'_\alpha}{\mu''_\alpha} \quad \text{and} \quad \lim_{t\to\infty} P''_{i\alpha}(t) = \frac{\lambda''_\alpha}{\mu'_\alpha}. \tag{3.1}$$

Theorem 3.1. *Let in a random fuzzy queuing system $RF/RF/k/FCFS/\infty/\infty$, the distributions of $\xi_i(\theta)$ and $\eta_i(\gamma)$ be non-lattice, and let the fuzzy variables λ_i and μ_i, $i = 1, 2\dots$, be continuous at the point $\alpha \in (0,1]$. Also, let the k servers work independently, then we have*

$$\lim_{t\to\infty} Ch\{\textit{all of } k \textit{ servers are busy at time } t\} = (E[\frac{\lambda}{\mu}])^k. \tag{3.2}$$

Proof. From Definition 10 and Proposition 1. for ith server, $i = 1, 2, \dots, k$, we have

$$Ch\{\textit{the ith server is busy at time } t\} = \int_0^1 Cr\{\theta \in \Theta | P_i(t)(\theta) \geq p\}dp$$

$$= \int_0^{\infty} Cr\{\theta \in \Theta | P_i(t)(\theta) \geq p\}dpE[P_i(t)] = \frac{1}{2}\int_0^1 (P'_{i\alpha}(t) + P''_{i\alpha}(t))dp.$$

It follows from the definition of the limit that there exist two non-negative real numbers t_1 and t_2, such that for any $t \geq t_1$

$$0 \leq P'_{i\alpha}(t) \leq \frac{\lambda'_\alpha}{\mu''_\alpha}.$$

and for any $t \geq t_2$

$$0 \leq P''_{i\alpha}(t) \leq \frac{\lambda''_\alpha}{\mu'_\alpha}.$$

Therefore, for any $t \geq \max(t_1, t_2)$, we have

$$0 \leq P'_{i\alpha}(t) + P''_{i\alpha}(t) \leq 2 + \frac{\lambda'_\alpha}{\mu''_\alpha} + \frac{\lambda''_\alpha}{\mu'_\alpha}.$$

Since $E[\frac{\lambda}{\mu}]$ is finite, $2 + \frac{\lambda'_\alpha}{\mu''_\alpha} + \frac{\lambda''_\alpha}{\mu'_\alpha}$ is an integrable function of α. Hence, we can apply Fatou's lemma, to conclude that

$$\liminf_{t\to\infty} \int_0^1 (P'_{i\alpha}(t) + P''_{i\alpha}(t))d\alpha \geq \int_0^1 \liminf_{t\to\infty} (P'_{i\alpha}(t) + P''_{i\alpha}(t))d\alpha.$$

and

$$\limsup_{t\to\infty} \int_0^1 (P'_{i\alpha}(t) + P''_{i\alpha}(t))d\alpha \leq \int_0^1 \limsup_{t\to\infty} (P'_{i\alpha}(t) + P''_{i\alpha}(t))d\alpha.$$

Next, since $\lambda'_\alpha, \lambda''_\alpha, \mu'_\alpha, \mu''_\alpha$ are almost surely continuous at the point α, by Lemma 1 we have

$$\lim_{t\to\infty} Ch\{\textit{the ith server is busy at time } t\}$$

$$= \frac{1}{2}\lim_{t\to\infty} \int_0^1 (P'_{i\alpha}(t) + P''_{i\alpha}(t))dp = \frac{1}{2}\int_0^1 \lim_{t\to\infty} (P'_{i\alpha}(t) + P''_{i\alpha}(t))dp$$

$$= \frac{1}{2}\int_0^1 (\frac{\lambda'_\alpha}{\mu''_\alpha} + \frac{\lambda''_\alpha}{\mu'_\alpha})d\alpha = E[\frac{\lambda}{\mu}].$$

Now, for k servers that work independently and are identically distributed, we have

$$\lim_{t\to\infty} Ch\{all\ of\ k\ servers\ are\ busy\ at\ time\ t\}$$

$$= \prod_{i=1}^{k} E[\frac{\lambda}{\mu}] = (E[\frac{\lambda}{\mu}])^k,$$

and the result follows. Theorem 1 is proved. □

Remark 3.1. *If X is the number of servers that are busy at time t, then X is a random variable with binomial distribution, that is, $X \sim B(k, Ch)$, where k is the number of servers and Ch is the average chance of the servers that are busy at time t. So, the average chance of the r servers ($r \le k$) that are busy at time t is given by*

$$\lim_{t\to\infty} Ch\{r\ servers\ are\ busy\ at\ time\ t\} = \binom{k}{r}(E[\frac{\lambda}{\mu}])^r(1 - E[\frac{\lambda}{\mu}])^{k-r}, \tag{3.3}$$

for $r = 0, 1, 2, ..., k$. Also, we have

$$\lim_{t\to\infty} Ch\{at\ least\ r\ servers\ are\ busy\ at\ time\ t\} = \sum_{i=r}^{k}\binom{k}{i}(E[\frac{\lambda}{\mu}])^i(1-E[\frac{\lambda}{\mu}])^{k-i}, \tag{3.4}$$

Therefore, the mean of the number of servers that are busy at time t is $kE[\frac{\lambda}{\mu}]$.

Then, the average chance of all of k servers are idle at time t is given by

$$\lim_{t\to\infty} Ch\{all\ of\ k\ servers\ are\ idle\ at\ time\ t\} = (1 - E[\frac{\lambda}{\mu}])^k. \tag{3.5}$$

4. The fuzzy simulation approach

Y.Liu and B.Liu [12] designed a fuzzy simulation procedure for both discrete and continuous cases.

(a) Discrete fuzzy vector: assume that f is a function, and $\xi = (\xi_1, ..., \xi_m)$ is a discrete fuzzy vector whose joint credibility distribution function is defined by

$$\mu_\xi(u) = \begin{cases} \mu_1, & u = u_1 \\ \mu_2, & u = u_2 \\ \cdots & \\ \mu_n, & u = u_n, \end{cases} \tag{4.1}$$

where $\mu_u = \min_{1\le i\le m} \mu^{(i)}(u_i)$, $u = (u_1, ..., u_m) \in \Re^m$ and $\mu^{(i)}$ are the credibility distribution functions of ξ_i for $i = 1, 2, ..., m$.

Let $a_i = f(u_i)$. Without loss of generality, we can assume that $a_1 \leq a_2 \leq \ldots \leq a_n$. Then the expected value is given by

$$E[f(\xi)] = \sum_{i=1}^{n} a_i p_i, \tag{4.2}$$

where

$$p_i = 1/2[\vee_{j=i}^{n} \mu_j - \vee_{j=i+1}^{n+1} \mu_j] + 1/2[\vee_{j=1}^{i} \mu_j - \vee_{j=0}^{i-1} \mu_j], \tag{4.3}$$

for $i = 1, 2, \ldots, n$ and $\mu_0 = \mu_{n+1} = 0$.

(b) Continuous fuzzy vector: assume that ξ is a continuous fuzzy vector with a credibility distribution function μ. In this case, the expected value can be estimated by formula (16).

5. Experiential results

In this section, we present some practical applications of the model under study, to show how the fuzzy simulation method can be used to estimate the average chance.

Example 1. Consider an investment bank with k servers. Let the interarrival times of customers be fuzzy variables with exponential distributions with $\lambda = (1/2/3)$ in minutes, and let the service times be fuzzy variables with exponential distributions with $\mu = (3/4/5)$ in minutes for k servers. Calculate the average chance of the event "all of k servers are busy at time t".

We use Theorem 1 and the simulation method, described in Section 4, to estimate the expectation $E[\frac{\lambda}{\mu}]$. The corresponding simulation results are shown in Table 1 and Figure 1, which contain the average chance of the event "all of k servers are busy at time t"for the number of different servers. The algorithm for simulating follows.

The Algorithm

1. **Generate** the random numbers $e_1(i)$ in the interval (1,3), and the random numbers $e_2(i)$ in the interval (3,5), $i = 1, 2, \ldots, n$.

2. **Set** $x_i = \frac{e_1(i)}{e_2(i)}$.

3. **Set** $\mu(i) = \min(\mu_1(i), \mu_2(i))$.

If x_i and x_j have the same values, remove x_j from the list of results, and set $\mu_i = \max(\mu_i, \mu_j)$.

4. **Apply** formulas (16) and (17).

5. **Calculate** E^k.

Table 1 and Figure 1 show that whenever the number of servers is bigger, the average chance of the event "all of k servers are busy at time t"is smaller, and it tends to zero as the number of servers increases. Notice that the results are obtained without using the α-cuts and we simulate the average chance. Also, the simulation procedure is stable in high repetitions and it is close to the true answer and it can not be invoked in few repetitions.

k	100	500	1000	10000	20000	30000
5	0.0318	0.0364	0.0398	0.0411	0.0414	0.0414
10	0.0012	0.0014	0.0016	0.0017	0.0018	0.0018
15	3.9172e-005	4.2981e-005	6.1171e-005	6.4359e-005	6.7551e-005	6.7551e-005
20	3.9523e-007	1.8686e-006	2.3734e-006	2.9698e-006	3.0270e-006	3.0270e-006

Table 1: The results of simulation of average chance for Example 1.

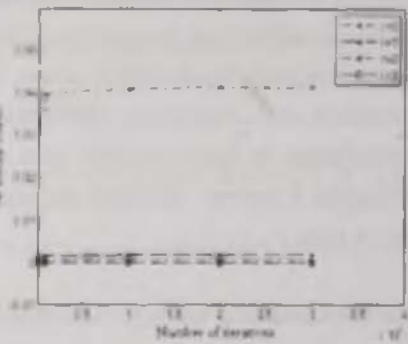

Figure 1: Convergence of the fuzzy simulation for Example 1.

Example 2. Let a bank have status customer. Also, let the interarrival times of costumers be fuzzy variables with exponential distribution with $\lambda = (2/3/4)$ in minutes, and the service times are fuzzy variables with exponential distribution with $\mu = (3/4/5)$ in minutes for any server. We want to calculate the average chance of the event "the number of different servers that are busy at time t".

We use Theorem 3.1, Remark 3.1 and the simulation method, described in Section 4, to estimate the average chance of the event "r servers ($r \leq k$) are busy at time t". The corresponding results of simulation are shown in Table 2 and in Figures 2-4. In Table 2, k is the number of iterations, r is the number of servers that are busy at time t, the "Error" rows contain the errors of the real solutions and simulated solutions, and the "Average Chance" rows contain the average chance of the event "the r servers out of 5 servers are busy at time t^r, $r = 0 - - -5$. Then we calculate the error of simulation. Similar to Example 1, from the results of Table 2 and Figure 2, we infer that whenever the number of iterations is getting bigger the simulation becomes closer to the real solution, and the fuzzy simulation procedure is stable in high repetitions. The Figures 3 and 4 show the error of simulation for different number of servers (out of 5 servers) that are busy at time t for $n = 10000, 20000$ and $n = 30000, 40000$, respectively. It is clear that the error of simulation tends to zero for $n > 40000$.

	n	500	1000	10000	20000	30000	40000
r=0	Average Chance	8.1376e-004	6.5993e-004	4.98[illegible]e-004	4.7863e-004	4.3809e-004	4.2924e-004
	Error	3.8453e-004	2.3069e-004	6.8879e-005	4.9386e-005	8.8500e-006	0.0000
r=1	Average Chance	0.0164	0.0109	0.0087	0.0085	0.0083	0.0080
	Error	0.0064	0.0029	6.8956e-004	4.9354e-004	2.9215e-004	0.0000
r=2	Average Chance	0.0723	0.0645	0.0617	0.0603	0.0593	0.0592
	Error	0.0131	0.0053	0.0025	0.0011	1.0000e-004	0.0000
r=3	Average Chance	0.2363	0.2334	0.2339	0.2211	0.2205	0.2200
	Error	0.0163	0.0134	0.0039	0.0011	5.0000e-004	0.0000
r=4	Average Chance	0.3981	0.4024	0.4079	0.4081	0.4085	0.4087
	Error	0.0106	0.0063	7.6731e-004	6.0000e-004	2.0000e-004	0.0000
r=5	Average Chance	0.2487	0.2684	0.2981	0.2996	0.3015	0.3036
	Error	0.0549	0.0352	0.0055	0.0040	0.0021	0.0000

Table 2: Results of the simulation of average chance for Example 2.

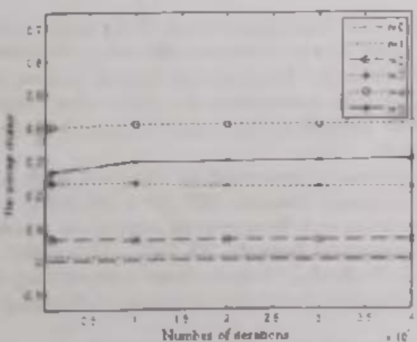

Figure 2: Convergence of the fuzzy simulation for Example 2.

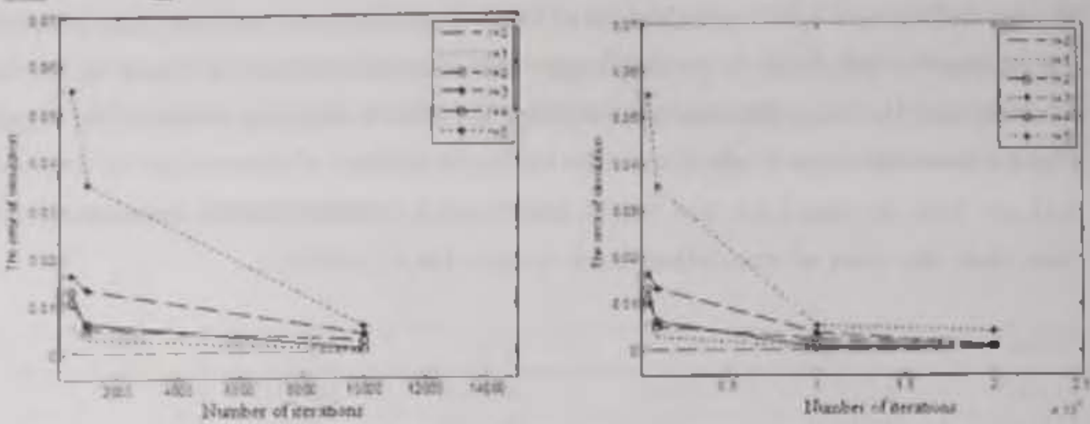

Figure 3: Convergence of fuzzy simulation for $n = 10000, 20000$ for Example 2.

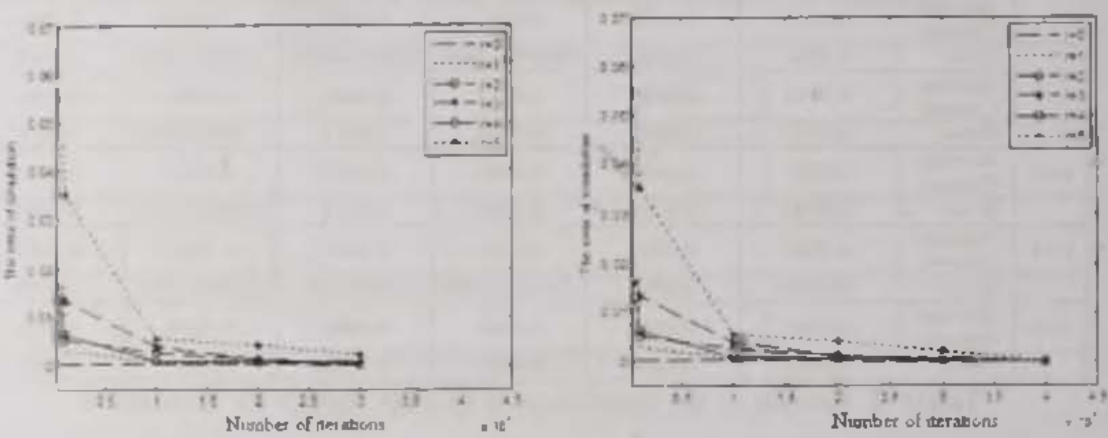

Figure 4: Convergence of fuzzy simulation for $n = 30000, 40000$ for Example 2.

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