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UNIQUENESS OF MEROMORPHIC FUNCTIONS SHARING ONE VALUE OR FIXED POINTS

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Abstract. [1] In this paper we study the uniqueness problems on meromorphic functions sharing a nonzero finite value or fixed points. Our results improve or generalize those given by Fang and Hua [7], Yang and Hua [18], Fang and Qiu [9], Cao and Zhang [2].

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Keywords: Uniqueness; meromorphic function; sharing value; fixed point.

1. Introduction and main results

Let $\mathbb{C}$ denote the complex plane and let $f(z)$ be a non-constant meromorphic function defined on $\mathbb{C}$. We assume that the reader is familiar with the standard notions used in the Nevanlinna value distribution theory, such as $T(r,f)$, $m(r,f)$, $N(r,f)$ (see [10, 12, 19, 20]). Let $S(r,f)$ denote any quantity that satisfies the condition $S(r,f) = o(T(r,f))$ as $r \to \infty$ outside possible an exceptional set of finite linear measure. A meromorphic function $a(z)$ is called a small function of $f(z)$ if $T(r,a) = S(r,f)$.

Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, and let $a(z)$ be a small function of $f(z)$ and $g(z)$. We say that $f(z)$ and $g(z)$ share $a(z)$ CM (counting multiplicities) if $f(z) - a(z)$ and $g(z) - a(z)$ have the same zeros with the same multiplicities, and we say that $f(z)$ and $g(z)$ share $a(z)$ IM (ignoring multiplicities) if the multiplicities are ignored. We denote by $N_{k)}(r, \frac{1}{f-a})$ (or $\overline{N}_{k)}(r, \frac{1}{f-a})$) the counting function for zeros of $f - a$ with multiplicity $\le k$ (ignoring multiplicities), and by $N_{(k}(r, \frac{1}{f-a})$ (or $\overline{N}_{(k}(r, \frac{1}{f-a})$) the counting function for zeros of $f-a$ with multiplicity $\ge k$ (ignoring multiplicities). Also, we set

$$N_k(r, \frac{1}{f-a}) = \overline{N}(r, \frac{1}{f-a}) + \overline{N}_{(2}(r, \frac{1}{f-a}) + \overline{N}_{(3}(r, \frac{1}{f-a}) + \cdots + \overline{N}_{(k}(r, \frac{1}{f-a}).$$

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We say that a finite value z_0 is a fixed point of f if $f(z_0) = z_0$ or z_0 is a zero of $f(z) - z$.

The following theorem is well known in the value distribution theory (see [1, 3]).

Theorem A. *Let $f(z)$ be a transcendental meromorphic function, and let $n \geq 1$ be a positive integer. Then $f^n f' = 1$ has infinitely many solutions.*

Fang and Hua [7], and Yang and Hua [18], respectively have obtained a unicity theorem corresponding to Theorem A.

Theorem B. *Let f and g be two non-constant entire (resp., meromorphic) functions, and let $n \geq 6$ (resp., $n \geq 11$) be a positive integer. If $f^n(z)f'(z)$ and $g^n(z)g'(z)$ share 1 CM, then either $f(z) = c_1 e^{cz}$ and $g(z) = c_2 e^{-cz}$, where c_1, c_2 and c are three constants satisfying $4(c_1c_2)^{n+1}c^2 = -1$, or $f(z) \equiv tg(z)$ for a constant t such that $t^{n+1} = 1$.*

Corresponding to the uniqueness of entire or meromorphic functions sharing fixed points, Fang and Qiu [9] obtained the following result.

Theorem C. *Let f and g be two non-constant meromorphic (resp., entire) functions, and let $n \geq 11$ (resp., $n \geq 6$) be a positive integer. If $f^n(z)f'(z)$ and $g^n(z)g'(z)$ share z CM, then either $f(z) = c_1 e^{cz^2}$ and $g(z) = c_2 e^{-cz^2}$, where c_1, c_2 and c are three constants satisfying $4(c_1c_2)^{n+1}c^2 = -1$, or $f(z) \equiv tg(z)$ for a constant t such that $t^{n+1} = 1$.*

For more results in this direction, we refer the reader to [4] — [9], [11],[13] – [16], [18], [21] – [24]. Cao and Zhang [2] extended Theorems B and C as follows.

Theorem D. *Let $f(z)$ and $g(z)$ be two transcendental meromorphic functions, whose zeros are of multiplicities at least k, where k is a positive integer, and let $n > \max\{2k-1, k+4/k+4\}$ be a positive integer. If $f^n f^{(k)}$ and $g^n g^{(k)}$ share z CM, and f and g share ∞ IM, then one of the following two conclusions holds:*

(1) $f^n f^{(k)} = g^n g^{(k)}$;

(2) $f = c_1 e^{cz^2}$, $g = c_2 e^{-cz^2}$, where c_1, c_2, c are constants such that $4(c_1c_2)^{n+1}c^2 = -1$.

Theorem E. *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, whose zeros are of multiplicities at least k, where k is a positive integer, and let $n > \max\{2k-1, k+4/k+4\}$ be a positive integer. If $f^n f^{(k)}$ and $g^n g^{(k)}$ share 1 CM, and f and g share ∞ IM, then one of the following two conclusions holds:*

(1) $f^n f^{(k)} = g^n g^{(k)}$;

(2) $f = c_3 e^{dz}$, $g = c_4 e^{-dz}$, where c_3, c_4, d are constants such that $(-1)^k(c_3c_4)^{n+1}d^{2k} = 1$.

In this paper we show that in Theorems D and E the condition "f and g share ∞ IM" can be removed. Specifically we prove the following results.

Theorem 1.1. *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions with $\sigma(f) < +\infty$, whose zeros are of multiplicities at least k, where k is a positive integer, and let $n > \max\{2k+1, 2(\sigma(f)-1)k-3, k+4/k+8\}$ be a positive integer. If $f^n f^{(k)}$ and $g^n g^{(k)}$ share z CM, then one of the following two conclusions holds:*
(1) $f^n f^{(k)} = g^n g^{(k)}$;
(2) $f = c_1 e^{cz^2}$, $g = c_2 e^{-cz^2}$, where c_1, c_2 and c are constants such that $4(c_1c_2)^{n+1}c^2 = -1$.

Theorem 1.2. *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions with $\sigma(f) < +\infty$, whose zeros are of multiplicities at least k, where k is a positive integer, and let $n > \max\{2k-1, 2(\sigma(f)-1)k-1, k+4/k+5\}$ be a positive integer. If $f^n f^{(k)}$ and $g^n g^{(k)}$ share 1 CM, then one of the following two conclusions holds:*
(1) $f^n f^{(k)} = g^n g^{(k)}$;
(2) $f = c_3 e^{dz}$, $g = c_4 e^{-dz}$ where c_3, c_4, d are constants such that $(-1)^k (c_3c_4)^{n+1} d^{2k} = 1$.

To prove Theorems 1.1 and 1.2, we need the following results.

Proposition 1.1. *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions with $\sigma(f) < +\infty$, and let n and k be two positive integers such that $n > \max\{2k+1, 2(\sigma(f)-1)k-3\}$. If $f^n f^{(k)} g^n g^{(k)} = z^2$, then $f = c_1 e^{cz^2}$, $g = c_2 e^{-cz^2}$, where c_1, c_2 and c are constants such that $4(c_1c_2)^{n+1}c^2 = -1$.*

Proposition 1.2. *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions with $\sigma(f) < +\infty$, and let n and k be two positive integers such that $n > \max\{2k-1, k+1, 2(\sigma(f)-1)k-1\}$. If $f^n f^{(k)} g^n g^{(k)} = 1$, then $f = c_3 e^{dz}$, $g = c_4 e^{-dz}$, where c_3, c_4 and d are constants such that $(-1)^k (c_3c_4)^{n+1} d^{2k} = 1$.*

2. Preliminary lemmas

Lemma 2.1 (see [19]). *Let $f(z)$ be a non-constant meromorphic function and let $a_0(z), a_1(z), \ldots, a_n(z)\ (\not\equiv 0)$ be small functions of f. Then*

$$T(r, a_n f^n + a_{n-1} f^{n-1} + \cdots + a_0) = nT(r,f) + S(r,f).$$

Lemma 2.2 ([19], p. 21). *Let $f(z)$ be a non-constant meromorphic function in the complex plane. If the order of $f(z)$ is finite, then*

$$m(r, \frac{f'}{f}) = O(\log r), \quad r \to \infty.$$

Lemma 2.3 ([19], p. 65). *Let $h(z)$ be a non-constant entire function and let $f(z) = e^{h(z)}$. Let λ and μ be the order and the lower order of $f(z)$, respectively. We have*
(i) *If $\mu < \infty$, then μ is a positive integer, $h(z)$ is a polynomial of degree μ, and $\lambda = \mu$.*
(ii) *If $\mu = \infty$, then $h(z)$ is transcendental and $\lambda = \mu$.*

Lemma 2.4. *Let $f(z)$ be a non-constant meromorphic function of finite order, and let k be a positive integer. Suppose that $f^{(k)} \not\equiv 0$, then*

$$N(r, \frac{1}{f^{(k)}}) \le N(r, \frac{1}{f}) + k\overline{N}(r, f) + O(\log r).$$

Proof. Since f is of finite order, by Lemma 2.2, we have

$$m(r, \frac{f'}{f}) = O(\log r).$$

Now we use mathematical induction to prove that $m(r, \frac{f^{(k)}}{f}) = O(\log r)$. Suppose that the conclusion is true for $k = m$. For $k = m+1$ we have

$$\frac{f^{(m+1)}}{f} = (\frac{f^{(m)}}{f})' + \frac{f^{(m)}}{f}\frac{f'}{f}.$$

Then we can write

$$\begin{aligned} m(r, \frac{f^{(m+1)}}{f}) &\le m(r, (\frac{f^{(m)}}{f})') + m(r, \frac{f^{(m)}}{f}) + m(r, \frac{f'}{f}) + O(1) \\ &= m(r, \frac{(\frac{f^{(m)}}{f})'}{\frac{f^{(m)}}{f}}\frac{f^{(m)}}{f}) + O(\log r) \le m(r, \frac{(\frac{f^{(m)}}{f})'}{\frac{f^{(m)}}{f}}) + m(r, \frac{f^{(m)}}{f}) + O(\log r) \\ &= O(\log r). \end{aligned}$$

Moreover, we have

$$m(r, \frac{1}{f}) \le m(r, \frac{1}{f^{(k)}}) + m(r, \frac{f^{(k)}}{f}) = m(r, \frac{1}{f^{(k)}}) + O(\log r).$$

Hence

$$T(r, f) - N(r, \frac{1}{f}) \le T(r, f^{(k)}) - N(r, \frac{1}{f^{(k)}}) + O(\log r).$$

Therefore

$$\begin{aligned} N(r,\frac{1}{f^{(k)}}) &\leq T(r,f^{(k)}) - T(r,f) + N(r,\frac{1}{f}) + O(\log r) \\ &= m(r,f^{(k)}) + N(r,f^{(k)}) - T(r,f) + N(r,\frac{1}{f}) + O(\log r) \\ &\leq m(r,f) + m(r,\frac{f^{(k)}}{f}) + N(r,f) + k\overline{N}(r,f) - T(r,f) + N(r,\frac{1}{f}) + O(\log r) \\ &= N(r,\frac{1}{f}) + k\overline{N}(r,f) + O(\log r). \end{aligned}$$

This completes the proof of Lemma 2.4.

Lemma 2.5 ([18]). *Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, n and k be two positive integers, and a be a finite nonzero constant. If f and g share a CM, then one of the following conclusions holds:*
(i) $T(r,f) \leq N_2(r,1/f) + N_2(r,1/g) + N_2(r,f) + N_2(r,g) + S(r,f) + S(r,g)$, the same inequality holding for $T(r,g)$;
(ii) $fg \equiv a^2$;
(iii) $f \equiv g$.

3. Proofs of Propositions 1.1–1.2

Proof of Proposition 1.1. We first prove that

$$f \neq 0, \quad g \neq 0. \tag{3.1}$$

We have

$$f^n f^{(k)} g^n g^{(k)} = z^2. \tag{3.2}$$

Suppose that $z_0 \neq 0$ is a zero of f, say of multiplicity l, then z_0 is a pole of g, say of multiplicity s. Then we have $nl + l - k = ns + s + k$, implying $(n+1)(l-s) = 2k$, which is impossible since by assumption $n > 2k+1$.
Now suppose that $z = 0$ is a zero of f, say of multiplicity l_1. If $z = 0$ is not a pole of g, then $z = 0$ must be the zero of z^2 of multiplicity $nl_1 + l_1 - k > 2$, which is a contradiction. If $z = 0$ is a pole of g, say of multiplicity s_1, then we have

$$(n+1)(l_1 - s_1) = 2k + 2,$$

which is impossible since by assumption $n > 2k+1$. So f has no zeros. Similarly, it can be shown that g also has no zeros. Thus (3.1) is proved.
Next, we prove that

$$N(r,f) = O(\log r), \quad N(r,g) = O(\log r). \tag{3.3}$$

To this end, we rewrite (3.2) as follows

$$f^n f^{(k)} = \frac{z^2}{g^n g^{(k)}}. \tag{3.4}$$

From (3.4) we deduce that

$$N(r, f^n f^{(k)}) = N(r, \frac{1}{g^n g^{(k)}}). \tag{3.5}$$

Since $N(r, f^n f^{(k)}) = (n+1)N(r, f) + k\overline{N}(r, f)$, using (3.5) and Lemma 2.4, we obtain

$$(n+1)N(r, f) + k\overline{N}(r, f) \le k\overline{N}(r, g) + O(\log r). \tag{3.6}$$

Similarly we get

$$(n+1)N(r, g) + k\overline{N}(r, g) \le k\overline{N}(r, f) + O(\log r). \tag{3.7}$$

A combination of (3.6) and (3.7) yields

$$N(r, f) + N(r, g) = O(\log r). \tag{3.8}$$

Thus we obtain (3.3), which means that both f and g have at most finitely many poles. Now we prove that

$$\sigma(f) = \sigma(g). \tag{3.9}$$

It is easy to show that both f and g must be transcendental meromorphic functions. Note that $nT(r, f) = T(r, f^n) =$

$$= T(r, f^n f^{(k)}/f^{(k)}) \le T(r, f^n f^{(k)}) + (k+1)T(r, f) + S(r, f), \tag{3.10}$$

and $T(r, f^n f^{(k)}) = T(r, \frac{z^2}{g^n g^{(k)}}) \le$

$$\le T(r, g^n g^{(k)}) + S(r, g) \le (n+k+1)T(r, g) + S(r, g). \tag{3.11}$$

Combining (3.10) and (3.11) we get

$$(n-k-1)T(r, f) \le (n+k+1)T(r, g) + S(r, f) + S(r, g).$$

Since $n > 2k+1$, we have $T(r, f) = O(T(r, g))$. Similarly we obtain $T(r, g) = O(T(r, f))$. Thus (3.9) is proved. Note that $\sigma(f) < +\infty$. Let

$$f = \frac{e^{h(z)}}{p(z)}, \quad g = \frac{e^{h_1(z)}}{q(z)},$$

where $p(z)$ and $q(z)$ are polynomials with $\deg(p(z)) = p$ and $\deg(q(z)) = q$, while $h(z)$ and $h_1(z)$ are non-constant entire functions. By Lemma 2.3, $h(z)$ and $h_1(z)$ are polynomials with $\deg(h(z)) = \deg(h_1(z)) = h = \sigma(f)$. Then we have

$$f^n = \frac{e^{nh(z)}}{p^n(z)}, \quad g^n = \frac{e^{nh_1(z)}}{q^n(z)}.$$

By mathematical induction we get

$$f^n f^{(k)} = \frac{e^{(n+1)h(z)}P_k(z)}{p^{n+k+1}(z)}, \quad g^n g^{(k)} = \frac{e^{(n+1)h_1(z)}Q_k(z)}{q^{n+k+1}(z)},$$

where $P_k(z)$ and $Q_k(z)$ are two polynomials with $\deg(P_k(z)) = k(h-1+p)$ and $\deg(Q_k(z)) = k(h-1+q)$. By (3.2), we get $h(z)+h_1(z) \equiv C$, where C is a constant. Furthermore, we have

$$\deg(P_k(z)) + \deg(Q_k(z)) = \deg(p^{n+k+1}(z)) + \deg(q^{n+k+1}(z)) + 2,$$

implying that

$$2k(h-1) = (n+1)(p+q) + 2. \tag{3.12}$$

By (3.8), if $N(r,f)+N(r,g) \neq 0$, then $p+q \geq 1$, and from (3.12) we obtain $n \leq 2k(h-1)-3 = 2k(\sigma(f)-1)-3$, which contradicts the assumption that $n > 2k(\sigma(f)-1)-3$. Therefore $N(r,f)+N(r,g) = 0$, showing that both f and g are entire functions and $p = q = 0$. From (3.12) we obtain that $h = 2$ and $k = 1$, and from (3.2) we have $h'(z) = l_2 z$, $h_1'(z) = l_3 z$ and $h(z) = cz^2 + l_3$, $h_1(z) = -cz^2 + l_4$. So, we can rewrite f and g as $f = c_1 e^{cz^2}$ and $g = c_2 e^{-cz^2}$, where c_1, c_2 and c are constants such that $4(c_1c_2)^{n+1}c^2 = -1$.

This completes the proof of Proposition 1.1.

Proof of Proposition 1.2. By the same reasoning as in the proof of Proposition 1.1, we get

$$2k(h-1) = (n+1)(p+q). \tag{3.13}$$

In view of $f^n f^{(k)} g^n g^{(k)} = 1$, if $N(r,f)+N(r,g) \neq 0$, then $p+q \geq 1$, and from (3.13) we obtain $n \leq 2k(h-1)-1 = 2k(\sigma(f)-1)-1$, which contradicts the assumption that $n > 2k(\sigma(f)-1)-1$. Therefore $N(r,f)+N(r,g) = 0$, showing that both f and g are entire functions and $p = q = 0$. From (3.13) we obtain that $h = 1$. Thus $h(z) = dz + l_5$ and $h_1(z) = -dz + l_6$. Finally, we rewrite f and g as $f = c_3 e^{dz}$ and $g = c_4 e^{-dz}$, where c_3, c_4 and d are nonzero constants, and deduce that $(-1)^k (c_3c_4)^{n+1} d^{2k} = 1$. This completes the proof of Proposition 1.2.

4. Proof of Theorem 1.1

Let $F = f^n f^{(k)}$, $G = g^n g^{(k)}$, $F^* = F/z$, and $G^* = G/z$. Then F^* and G^* share 1 CM. In view of Lemma 2.5, we consider three cases.

Case 1.

$$T(r,F^*) \leq N_2(r,1/F^*)$$

(4.1) $$+N_2(r,1/G^*)+N_2(r,F^*)+N_2(r,G^*)+S(r,f)+S(r,g).$$

We deduce from (4.1) that

$$T(r,F)\leq N_2(r,1/F)+N_2(r,1/G)+2\overline{N}(r,f)+2\overline{N}(r,g)+3\log r+S(r,f)+S(r,g).$$

Obviously,

$$N(r,F)=(n+1)N(r,f)+k\overline{N}(r,f)+S(r,f). \tag{4.2}$$

Also, we have

$$\begin{aligned} nm(r,f) &= m(r,F/f^{(k)})\leq m(r,F)+m(r,1/f^{(k)})+S(r,f)\\ &= m(r,F)+T(r,f^{(k)})-N(r,1/f^{(k)})+S(r,f)\\ &\leq m(r,F)+T(r,f)+k\overline{N}(r,f)-N(r,1/f^{(k)})+S(r,f). \end{aligned} \tag{4.3}$$

It follows from (4.2), (4.3) and Lemma 2.1 that

$$\begin{aligned} (n-1)T(r,f) &\leq T(r,F)-N(r,f)-N(r,1/f^{(k)})+S(r,f)\\ &\leq N_2(r,1/F)+N_2(r,1/G)+2\overline{N}(r,f)+2\overline{N}(r,g)\\ &\quad -N(r,f)-N(r,1/f^{(k)})+3\log r+S(r,f)+S(r,g)\\ &\leq 2\overline{N}(r,1/f)+2\overline{N}(r,1/g)+N(r,1/g)+k\overline{N}(r,g)\\ &\quad +\overline{N}(r,f)+2\overline{N}(r,g)+3\log r+S(r,f)+S(r,g)\\ &\leq \frac{2}{k}(T(r,f)+T(r,g))+(k+4)T(r,g)+3\log r+S(r,f)+S(r,g). \end{aligned} \tag{4.4}$$

Similarly we obtain

$$\begin{aligned} (n-1)T(r,g) &\leq \frac{2}{k}(T(r,f)\\ &+T(r,g))+(k+4)T(r,f)+3\log r+S(r,f)+S(r,g). \end{aligned} \tag{4.5}$$

Combining (4.4) and (4.5) we get

$$\begin{aligned} &(n-1)(T(r,f)+T(r,g))\\ &\leq (\frac{4}{k}+k+4)(T(r,f)+T(r,g))+6\log r+S(r,f)+S(r,g). \end{aligned} \tag{4.6}$$

Noting that $T(r,f)\geq \log r+O(1)$, $T(r,g)\geq \log r+O(1)$ and $n>k+4/k+8$, we get a contradiction from (4.6).

Case 2. We have $f^nf^{(k)}g^ng^{(k)}=z^2$, and by Proposition 1.1 we get conclusion (2) of the theorem 1.1.

Case 3. We have $f^nf^{(k)}=g^ng^{(k)}$. This completes the proof of Theorem 1.1.

The proof of Theorem 1.2 is similar to that of Theorem 1.1, the only difference is that instead of Proposition 1.1, we use Proposition 1.2.

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