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CYCLIC REPRESENTATIONS OF C^*-ALGEBRAS ON FOCK SPACE

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Abstract. [1] The paper characterizes C^*-algebras generated by semigroups of composition operators acting upon Fock space. With an approach of approximation from nonharmonic analysis, sufficient conditions for cyclic representations are obtained. We also characterize the cyclic representation of a C^*-algebra of measures generated by Heisenberg groups.

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1. Introduction

This paper is devoted to the study of properties of the C^*-algebras generated by semigroups of composition operators acting upon Fock space of $\mathbf{C}^n$, $n \geq 1$. Representations of a C^*-algebra of measures generated by Heisenberg groups are also considered.

The main novelty of the analysis carried out in this paper lies precisely in the fact that we analyze the cyclic representations of the C^*-algebras with the approach of uniqueness of analytic functions, which is crucial in solving approximation problems in nonharmonic analysis (see, [8], [16], [17]).

Semigroups appear in many areas of analysis (harmonic analysis, representation theory, operator theory, ergodic theory, etc.) The properties of semigroups of holomorphic flows have been extensively studied during the past several decades. Here we mention two known facts that are related to our work in this paper. In [4], Berkson, Kaufman and Porta proved the strong continuity of these flows on Hardy spaces. A complete description of semigroups of holomorphic flows on $\mathbf{C}$ was obtained in [11], by using an approach and techniques, which are quite different and independent of operator-theoretic considerations.

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It was Grothendieck (see [9]), who initiated the study of approximation properties of operator algebras associated with discrete groups, whose fundamental ideas have been applied to the study of groups. In this case one discovers that some important properties of groups can be expressed in terms of approximation properties of the associated operator algebras. Also, various important properties of the groups can be expressed in terms of analytic properties of these algebras. An illustration of nontrivial interaction between analytic and geometric properties of groups and a short survey of approximation properties of operators algebras associated with discrete groups can be found in [6].

Throughout the paper we use the following notation: the points of $\mathbb{C}^n$ are denoted by $z = (z_1, ..., z_n)$, where $z_k \in \mathbb{C}$. If $z_k = x_k + iy_k$, $x = (x_1, ..., x_n)$, $y = (y_1, ..., y_n)$, then we write $z = x + iy$. The vectors $x = \Re z$ and $y = \Im z$ are the real and imaginary parts of z, respectively. $\mathbb{R}^n$ stands for the set of all $z \in \mathbb{C}^n$ with $\Im z = 0$. Also, we denote

$$|z| = (|z_1|^2+\cdots+|z_n|^2)^{1/2},\ |\Re z| = (|x_1|^2+\cdots+|x_n|^2)^{1/2},\ |\Im z| = (|y_1|^2+\cdots+|y_n|^2)^{1/2},$$
$$z^\beta = z_1^{\beta_1}\cdots z_n^{\beta_n}, \quad \langle z, t\rangle = z_1 t_1 + \cdots + z_n t_n.$$

The Bargmann-Fock space $\mathcal{F}_n^2(\mathbb{C}^n)$ is defined to be the Hilbert space of entire functions on $\mathbb{C}^n$ equipped with the inner product:

$$\langle f, g\rangle = \frac{1}{(2\pi)^n}\int_{\mathbb{C}^n} f(z)\overline{g(z)}e^{-\frac{1}{2}|z|^2}dv(z),$$

where v denotes the n-dimensional Lebesgue measure on $\mathbb{C}^n$. The norm in $\mathcal{F}_n^2(\mathbb{C}^n)$ is defined by $\|f\| = \sqrt{\langle f, f\rangle}$ (see, [2], [3], [18]).

The reproducing kernel for the Fock space is given by $K_w(z) = e^{\langle z,w\rangle/2}$, where $\langle z, w\rangle = \sum_{j=1}^n z_j\overline{w_j}$. It is well known that $\|K_w\| = e^{|w|^2/4}$. The Bargmann-Fock spaces has been studied by many authors and it is rooted from mathematical problems of relativistic physics (see [15]) or from quantum optics (see [13]). In physics the Bargmann-Fock space contains the canonical coherent states, so it is the main tool for studying the bosonic coherent state theory of radiation field (see [14]).

The Bargmann-Fock spaces has also been proved invaluable in the theory of wavelets. In fact, the Bargmann transform is a unitary map from $L^2(\mathbb{R})$ onto the Bargmann-Fock space $\mathcal{F}_1^2(\mathbb{C})$, transforming the family of evaluation functionals at a point into canonical coherent states, which are nothing but the Gabor wavelets.

In the last years there was an increasing interest to the characterization of composition operators acting upon Fock space. For instance, bounded and compact composition operators acting upon Fock space $\mathcal{F}^2_n(\mathbb{C}^n)$ were described in [7]. In the recent paper [19], boundedness and compactness of densely defined operators on Fock space $\mathcal{F}^2_1(\mathbb{C})$ were characterized in terms of Berezin transform.

Motivated by [5], [6], [11], [18], [19], it is rather natural to study the approximation properties of C^*-algebras generated by semigroups of composition operators acting upon Fock space.

The paper is organized as follows. Section 2 is devoted to the study of composition operators on Fock space of $\mathbb{C}^n$ which induce holomorphic flows. In Section 3, we obtain sufficient conditions for representations of a C^*-algebra of composition flow to be cyclic in Fock space of $\mathbb{C}^n$. In Section 4, similar conditions are obtained for a C^*-algebra of measures generated by Heisenberg groups.

2. Holomorphic flows induced by a bounded composition operator on Fock space of $\mathbb{C}^n$

In this section we describe the holomorphic flows induced by a bounded composition operator on Fock space of $\mathbb{C}^n$. To this end, we first recall some basic definitions and results.

Let G be a domain in $\mathbb{C}^n$, and let $H(G)$ be the set of holomorphic functions on G. A one-parameter family $\varphi(t,z)$ of nonconstant functions from G to G satisfying $\varphi(0,z)=z$ and $\varphi(t+s,z)=\varphi(s,(t,z))$ for all $s,t\geq 0$ and $z\in G$ is called a semigroup flow (see [11]). The family $(C_{\varphi_t})_{t\geq 0}$ of composition operators on $H(G)$ is given by

$$(C_{\varphi_t}f)(z)=f(\varphi(t,z))$$

for every $t\geq 0$ and $f\in H(G)$. Notice that since $\varphi(t,z)$ is a flow, the semigroup property $C_{\varphi_{t+s}}=C_{\varphi_t}C_{\varphi_s}$ is satisfied.

The next result, which was established in [7], contains the boundedness and compactness of composition operator on Fock space of $\mathbb{C}^n$.

Lemma 2.1 ([7]). *Let $\varphi:\mathbb{C}^n\to\mathbb{C}^n$ be a holomorphic mapping. If for $f\in\mathcal{F}^2_n(\mathbb{C}^n)$, $C_\varphi(f):=f(\varphi(z))$ is bounded on $\mathcal{F}^2_n(\mathbb{C}^n)$, then $\varphi(z)=Az+B$, where A is an $n\times n$ matrix and B is an $n\times 1$ vector. Furthermore, $\|A\|\leq 1$, and if $|A\xi|=|\xi|$ for some $\xi\in\mathbb{C}^n$, then $\langle A\xi,B\rangle=0$; if C_φ is compact, then $\|A\|<1$.*

Conversely, let $\varphi(z)=Az+B$, where A is an $n\times n$ matrix and B is an $n\times 1$ vector. If $\langle A\xi,B\rangle=0$ whenever $|A\xi|=|\xi|$, then C_φ is bounded on $\mathcal{F}^2_n(\mathbb{C}^n)$; if $\|A\|<1$, then C_φ is compact on $\mathcal{F}^2_n(\mathbb{C}^n)$.

The main result of this section is the following theorem.

Theorem 2.1. *Suppose that $\varphi(t,z) : \mathbb{C}^n \to \mathbb{C}^n$ is an one-parameter family of holomorphic mapping satisfying $\varphi(0,z) = z$ and $\varphi(t+s,z) = \varphi(s,(t,z))$ for all $s,t \geq 0$ and $z \in \mathbb{C}^n$. If the family $(C_{\varphi_t})_{t\geq 0}$ is bounded on $\mathfrak{F}^2_n(\mathbb{C}^n)$, then $\varphi(t,z) = e^{Ft}z + (e^{Ft_0} - I)^{-1}(e^{Ft} - I)B$ or $\varphi(t,z) = z + Dt$, where F is an $n \times n$ matrix, and B and D are $n \times 1$ vectors.*

Proof. By Lemma 2.1 we have $\varphi(t,z) = A(t)z + B(t)$, where $A(t) = (a_{ij}(t))_{n\times n}$ is an $n \times n$ matrix satisfying $\|A(t)\| \leq 1$ and $B(t) = (b_{ij}(t))_{n\times 1}$ is an $n \times 1$ vector, $a_{ij}(t)$ and $b_{ij}(t)$ are differentiable functions of t.

From the semigroup property of the flow, we have

$$A(t+s)z + B(t+s) = A(t)(A(s)z + B(s)) + B(t) = A(t)A(s)z + A(t)B(s) + B(t).$$

Equating the coefficients of z, we get

$$A(t+s) = A(t)A(s) \tag{2.1}$$

and

$$B(t+s) = A(t)B(s) + B(t). \tag{2.2}$$

Since $\varphi(0,z) = z$, we have $A(0) = I$, where I is the unit matrix, and $B(0) = O$, where O is the zero vector. The differentiability of $A(t)$ and equality (2.1) imply $A(t) = e^{Ft}$, where F is an $n \times n$ matrix. Actually, we have

$$A'(t) = \lim_{\Delta t\to 0} \frac{A(t+\Delta t) - A(t)}{\Delta t} = A(t) \lim_{\Delta t\to 0} \frac{A(\Delta t) - A(0)}{\Delta t} = A(t)A'(0).$$

Next, the equality

$$A(t+s) = A(s)A(t) = A(t)A(s)$$

implies $A'(t) = A(t)A'(0) = A'(0)A(t)$. If $e^{Ft_0} \neq I$ for some $t_0 \geq 0$, then from (2.2) we have

$$B(t+s) = e^{Ft}B(s) + B(t), \tag{2.3}$$

$$B(s+t) = e^{Fs}B(t) + B(s). \tag{2.4}$$

Taking $s = t_0$ in (2.3) and (2.4), we get $B(t) = (e^{Ft_0} - I)^{-1}(e^{Ft} - I)B(t_0)$.

If $e^{Ft} \equiv I$, then from (2.3) we obtain $B(s+t) = B(t) + B(t)$. Thus, $B(t)$ is a continuous linear vector function in t with $B(0) = 0!$. So, we can conclude that $B(t) = Dt$ for some $n \times 1$ vector D in $\mathbb{C}^n$. This completes the proof of Theorem 2.1. □

3. Cyclic representations of C^*-algebras of composition flow

In this section we deal with the C^*-algebra:

$$\mathfrak{C}_{T_t} := C^*(\{C_{\varphi_t} : \varphi_t \in \Gamma\}),$$

where $C_{\varphi_t}(f(z)) = f(\varphi(t,z))$ and Γ is a discrete semigroup flow of $\mathbb{C}^n$.

We first recall some definitions from the theory of C^*-algebras (see, [1]). A bounded linear map $\pi : X \to Y$ between C^*-algebras X and Y is called a *-homomorphism if it preserves the algebraic operations and satisfies $\pi(x^*) = \pi(x)^*$ for any $x \in X$. A representation of a C^*-algebra $\mathfrak{C}$ is a *-homomorphism of $\mathfrak{C}$ into the C^*-algebra $\mathfrak{L}(H)$ of all bounded operators on some Hilbert space H. It is customary to refer the map $\rho : \mathfrak{C} \to \mathfrak{L}(H)$ as a representation of $\mathfrak{C}$ on H. An invariant subspace $\mathfrak{M}$ of the C^*-algebra $\rho(\mathfrak{C})$ is called a cyclic subspace if it contains a vector ξ, such that $\{\rho(\mathfrak{C})\xi,\ \xi \in H\}$ is dense in $\mathfrak{M}$. A representation ρ is called a cyclic representation if H itself is a cyclic subspace for ρ. Let Λ be a complex set and $f(z)$ be some holomorphic function. If $f(\Lambda) = 0$ implies $f(z) \equiv 0$, then Λ is called a *uniqueness set* for f.

The main result of this section is the following theorem.

Theorem 3.1. *Let $\Lambda = \{\lambda_k\}_{k=1}^{\infty}$ be a sequence of nonnegative real numbers. Suppose that $\rho(\varphi(t,z))$ is analytic on t and z separately, where $\varphi(t,z)$ is as in Theorem 2.1. Furthermore, suppose that $0 \le t \in \Lambda$ is the uniqueness set of some bounded holomorphic function in the right half plane. Then ρ is a cyclic representation of $\mathfrak{C}_{T_t}$ on $\mathcal{F}_n^2(\mathbb{C})$.*

Proof. To prove that ρ is a cyclic representation of the C^*-algebra $\mathfrak{C}_{T_t}$, it is enough to show that $\operatorname{span}\{\rho(\mathfrak{C}_{T_t})f\}$ is dense in $\mathcal{F}_n^2(\mathbb{C}^n)$ for some fixed $f \in \mathcal{F}_n^2(\mathbb{C}^n)$. Without loss of generality, let g be a function in $\mathcal{F}_n^2(\mathbb{C}^n)$ such that

$$|f(\rho(\varphi(t,z)))| = |g(\varphi(t,z))| \le e^{\alpha|z|^2}, \tag{3.1}$$

where the function $\varphi(t,z)$ is as in Theorem 2.1 and $\alpha < 1/2$ is some fixed positive constant. If the conditions of the theorem are satisfied, but $\operatorname{span}\{\rho(\mathfrak{C}_{T_t})f\}$ is not dense in $\mathcal{F}_n^2(\mathbb{C}^n)$, then by the Hahn-Banach Theorem, there exists a nontrivial bounded linear functional L which annihilates $\{\rho(\mathfrak{C}_{T_t})f\}$. Thus,

$$L(f(\rho(\varphi(t,z)))) = L(g(\varphi(t,z))) = 0.$$

Define

$$L(w) = \frac{1}{(2\pi)^n}\int_{\mathbb{C}^n} g(\varphi(w,z))\overline{q(z)}e^{-\frac{1}{2}|z|^2}\,dv(z),$$

and observe that $L(w)$ is an analytic function in the right half plane $\mathbb{C}_+ = \{w : \Re w > 0\}$. It follows from (3.1) that $L(w)$ is bounded in $\mathbb{C}_+$. Since by assumption $0 \le t \in \Lambda$ is the uniqueness set, we can conclude that $L(w) \equiv 0$. This completes the proof of Theorem 3.1. □

The following example illustrates Theorem 3.1.

Example. The mapping ρ defined by $\rho_t f = e^{-t} f$ is a cyclic representation of $\mathcal{C}_{T_t}$ on $\mathcal{F}_1^2(\mathbb{C})$. Indeed, in this case we take $f(z) = e^z$,

$$L(w) := \frac{1}{(2\pi)^n} \int_{\mathbb{C}^n} e^{ac^{-w}z + b\overline{q(z)}} e^{-\frac{1}{2}|z|^2} dv(z),$$

and

$$\Lambda = \left\{ \lambda_k : \lambda_k > 0, \sum_{k=1}^{\infty} \frac{\lambda_k}{1 + \lambda_k^2} = +\infty \right\}.$$

Then using the arguments of the proof of Theorem 3.1 and [10], we can conclude that $L(w) \equiv 0$, and the result follows.

4. Cyclicity of Segal-Bargmann representation.

In this section we study the cyclicity property of Segal-Bargmann representation. To this end, we first recall some basic facts on C^*-algebras generated by Heisenberg groups (see [5]). The Heisenberg group H_n is given by $\mathbb{C}^n \times \mathbb{R}$ with multiplication

$$(a, t)(b, s) = (a + b, s + t + \Im b.a/2),$$

where $\Im b.a/2 = (b.a - a.b)/2i$. It is well-known that the Lebesgue measure on $\mathbb{C}^n \times \mathbb{R}$ is bi-invariant Haar measure on H_n. In [5], Coburn focused on the Segal-Bargmann representation on Fock space. The representation is given by $\rho(a, t) = e^{it} W_a$, where

$$(W_a f)(z) = k_a(z) f(z - a)$$

and $k_a(z) = \exp\{\langle z, a\rangle - |a|^2/2\}$ is the normalized reproducing kernel. In representation theory, ρ is often extended to $M(H_n)$ and $L^1(H_n)$, the convolution algebra of bounded regular complex valued Borel measures on H_n and its closed two-sided ideal of measures that are absolutely continuous with respect to left Haar measure, respectively. It is represented as follows:

$$\rho(\sigma) = \int_{H_n} \rho(a, t) d\sigma(a, t). \tag{4.1}$$

The equation (4.1) determines an operator on $\mathcal{F}_n^2(\mathbb{C}^n)$ by

$$\langle \rho(\sigma) f, g\rangle = \int_{H_n} \langle \rho(a, t) f, g\rangle d\sigma(a, t),$$

where f and g are arbitrary functions in $\mathfrak{F}_n^2(\mathbb{C}^n)$ and $\langle\cdot,\cdot\rangle$ denotes the usual inner product on $L^2(\mathbb{C}^n)$.

The cut-down of ρ to $M(H_n)$, $L^1(H_n)$ is defined as follows:

$$\widetilde{\rho}(\sigma) = \int_{\mathbb{C}^n} W_a d\sigma(a). \tag{4.2}$$

Observe that $M(H_n)$ and $M(\mathbb{C}^n)$ are involution Banach algebras with $d\sigma^* = \overline{d\sigma(-a)}$. The twisted convolution $\tau\natural\sigma$ on $M(\mathbb{C}^n)$ is defined for any continuous on $\mathbb{C}^n$ function ϕ which vanishes at infinity as follows:

$$\int_{\mathbb{C}^n} \phi(a)d(\tau\natural\sigma)(a) = \int_{\mathbb{C}^n}\int_{\mathbb{C}^n} \phi(a+b)\chi_a(b/2)d\tau(a)d\tau(b),$$

where $\chi_a(z) = \exp\{i\Im\langle z,a\rangle\}$. Denote by $B(\mathbb{C}^n)$ the linear span of all continuous positive-definite functions on $\mathbb{C}^n$.

For bounded and continuous φ and $f\in\mathfrak{F}_n^2(\mathbb{C}^n)$, the Berezin-Toeplitz operator T_φ is defined by

$$(T_\varphi f)(z) = \frac{1}{(2\pi)^n}\int_{\mathbb{C}^n} \varphi(a)f(a)e^{\langle z,a\rangle/2}e^{-\frac{1}{2}|a|^2}dv(a).$$

In [5], it is proved that $\widetilde{\rho}$, defined by (4.2), is a faithful representation of $M(\mathbb{C}^n)_\natural$ on $\mathfrak{F}_n^2(\mathbb{C}^n)$. The following identities are also proved in [5]:

$$\mathfrak{C}^*\{\tilde{\rho}[M(\mathbb{C}^n)_\natural]\} = \mathfrak{C}^*\{\rho[M(H_n)]\} = \text{closure}\{T_\varphi:\ \varphi\in B(\mathbb{C}^n)\}.$$

Below we prove that both $\widetilde{\rho}$ and ρ are cyclic representations on $\mathfrak{F}_n^2(\mathbb{C}^n)$. To this end, we need the Jensen's formula for entire functions of several variables.

Let $\mathbf{e} = (e_1, e_2, ..., e_n)$ be a unit vector satisfying $e_j > 0$ $(j = 1, 2, ..., n)$. Denote by $N(\Lambda,\mathbf{e},t)$ the number of points of Λ lying on the segment $S(\mathbf{e},t) = \{(e_1\xi, e_2\xi, ..., e_n\xi) : |\xi|\le t\}$ of the complex affine straight line $S(\mathbf{e}) = \{(e_1\xi, e_2\xi, ..., e_n\xi) : \xi\in\mathbb{C}\}$. For an entire function $f(z)$, by $N(f,\mathbf{e},t)$ we denote the number of zeros of $f(z)$ in $S(\mathbf{e},t)$, counted according to their multiplicity. Also, we denote

$$S(r,\alpha) := \left(\frac{r_1}{r}e^{i\alpha_1}, \frac{r_2}{r}e^{i\alpha_2}, ..., \frac{r_{n-1}}{r}e^{i\alpha_{n-1}}, \frac{r_n}{r}\right),$$

where $r = (r_1, r_2, ..., r_n)$ satisfying $r_j > 0$ $(j = 1, 2, ..., n)$. For an entire function $f(z)$ $(f(0)\neq 0)$, the Jensen's formula is as follows (see [12], Chapter 4, Section 2.1-2.4):

$$\int_0^r \left(\frac{1}{t}\int_0^{2\pi}\cdots\int_0^{2\pi} N(f,S(r,\alpha),t)d\alpha_1...d\alpha_{n-1}\right)dt$$
$$= \frac{1}{2\pi}\int_0^{2\pi}\cdots\int_0^{2\pi}\log|f(r_1e^{i\theta_1},...,r_ne^{i\theta_n})|d\theta_1...d\theta_n - \log|f(0)|.$$

Theorem 4.1. *The representations ρ and $\widetilde{\rho}$ defined by (4.1) and (7), respectively, are cyclic representations of $M(H_n)$ and $M(\mathbb{C}^n)_\sharp$ on $\mathfrak{F}^2_n(\mathbb{C}^n)$, respectively.*

Proof. Since both of the closures of $M(H_n)$ and $M(\mathbb{C}^n)_\sharp$ under the discussed representations are equal to closure$\{T_\varphi : \varphi \in B(\mathbb{C}^n)\}$, it is enough to show that

$$\text{closure}\{T_\varphi : \varphi \in B(\mathbb{C}^n)\} = \mathfrak{F}^2_n(\mathbb{C}^n). \tag{4.3}$$

Let $\Lambda = \{\lambda_k\}_{k=1}^{\infty}$ be a sequence of nonnegative real numbers in $\mathbb{R}^n$, where $\lambda_k = (\lambda_k^1, \lambda_k^2, ..., \lambda_k^n)$. It is easy to see that the functions $\varphi_\lambda := e^{i\Im\langle\lambda,a\rangle/2} = e^{i\langle\lambda,\Im a\rangle/2}$, $\lambda \in \Lambda$ are in $B(\mathbb{C}^n)$. Thus, to prove (4.3), it is enough to show that the closure of linear span $\{T_{\varphi_\lambda} : \lambda \in \Lambda\}$ coincides with $\mathfrak{F}^2_n(\mathbb{C}^n)$ for a suitable selected sequence Λ. Denote

$$J(r,f) = \int_0^r \left(\frac{1}{t}\int_0^{2\pi} \cdots \int_0^{2\pi} N(f, S(r,\alpha), t)d\alpha_1 ... d\alpha_{n-1}\right)dt,$$

and observe that if Λ satisfies the condition

$$\limsup_{r\to+\infty} \frac{J(r,f)}{r^2} = +\infty, \tag{4.4}$$

then the closure of linear span $\{T_{\varphi_\lambda} : \lambda \in \Lambda\}$ coincides with $\mathfrak{F}^2_n(\mathbb{C}^n)$. Actually, if (4.4) is fulfilled, but the closure of linear span $\{T_{\varphi_\lambda} : \lambda \in \Lambda\} \neq \mathfrak{F}^2_n(\mathbb{C}^n)$, then by the Hahn-Banach Theorem, there exists a nontrivial bounded linear functional L which annihilates T_{φ_λ}, that is $L(T_{\varphi_\lambda} f) = 0$. Without loss of generality, let $f(z)$ be a function in $\mathfrak{F}^2_n(\mathbb{C}^n)$ satisfying

$$|f(z)| \le e^{-\varepsilon_0|z|^2}, \tag{4.5}$$

where ε_0 is some fixed positive number. Define

$$L(w) = \frac{1}{(2\pi)^n}\int_{\mathbb{C}^n} (T_{\varphi_w} f)(z)\overline{q(z)}e^{-\frac{1}{2}|z|^2}dv(z)$$
$$= \frac{1}{(2\pi)^{2n}}\int_{\mathbb{C}^n}\int_{\mathbb{C}^n} e^{i\langle w,\Im a\rangle} f(a)e^{\langle z,a\rangle/2}e^{-\frac{1}{2}|a|^2}dv(a)\overline{q(z)}e^{-\frac{1}{2}|z|^2}dv(z),$$

and observe that $L(w)$ is an entire function.

By (4.5), with some positive constant A_1 we have

$$|(T_{\varphi_w} f)(z)| \le A_1 e^{-\frac{1}{2\varepsilon_0}(|w|+|z|)^2}. \tag{4.6}$$

It follows from (4.6) that for sufficiently large r with some positive constant A_2 $\log|f(z)| \le A_2 r^2$. Hence by the Jensen's formula, we should have

$$\limsup_{r\to+\infty} \frac{J(r,f)}{r^2} < +\infty,$$

which contradicts (4.4). Thus, we conclude that $L(w) \equiv 0$, and the result follows. Theorem 4.1 is proved. □

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