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THE MATHEMATIZATION OF THE INDIVIDUAL SCIENCES - REVISITED

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ABSTRACT. We recall major findings of a systematic investigation of the mathematization of the individual sciences, conducted by the author in Bielefeld some 35 years ago under the direction of Klaus Krickeberg, and confront them with recent developments in physics, medicine, economics, and spectral geometry.

Dedicated to the 80th birthday of Klaus Krickeberg

INTRODUCTION - RECOLLECTIONS

In the years 1972-75, a small working group on *The mathematization of the individual sciences* was organized by Klaus Krickeberg and myself at the then young West-German university of Bielefeld. The group included representatives of all the disciplines represented at the university. Unfortunately, the engineering and the medical sciences were not represented, so that the important questions of mathematization of industrial production and health care were not addressed.

To my knowledge it was the first systematic investigation of the mathematization as seen from the individual sciences. The term *mathematization* was directed both to the shining modelling aspects (= selecting, finding, inventing the *right* specification, equations and set-up) and the rather profane tasks of executing the necessary calculations (= parameter estimations, analytic solutions, numerical simulations, approximations, stability/deformation/robustness arguments, and geometric interpretations). The investigation remained a singularity in its multi-disciplinary, almost all-embracing ambition. It was based on extensive literature study and a long series of hearings and interviews. It was, and perhaps continues to be an offence against two traditional imperatives of scientific research: *Don't mix*! and *Originality first*! We were continuously comparing - and confronting - different sciences with each other and their mathematization experiences, looking forth and back, mixing continuously mathematical, methodological, epistemological, educational, and ethical questions. More offending, we were not interested in the most recent top-notch results, but in the *relevance* of the applied mathematical methods for advances in the individual sciences, and in the question what the cases were *representative* for.

On the initiative of the wizard Alexander Ostrowski (1893-1986), Krickeberg and I collected some of our findings in a volume and published it with Birkhäuser in Basel in 1976, [10], see also [8] for a summary report.

Acknowledgment. For the following recollections and generalizations, I must take the responsibility alone, assuming that most of the views were or will be shared by Klaus Krickeberg. The findings are also based on later modelling experiences and continuing discussions with students and colleagues there. In particular, I am indebted to Viggo Andreasen, Peder Voetmann Christiansen, Niels Langager Ellegaard, Jens Høyrup, Jens Højgaard Jensen, Bent C. Jørgensen, Jesper Larsen, Anders Madsen, Mogens Niss, Johnny Ottesen, and Stig Andur Pedersen (all Roskilde), Philip J. Davis (Providence), Giampiero Esposito (Napoli), Roman Galar (Wroclaw), Martin Koch (Copenhagen), Matthias Lesch (Bonn), Glen Pate (Hamburg) and Erik Renström (Lund).

1. CONTINUING CHALLENGES

The use of mathematical arguments, first in pre-scientific investigations, then in other sciences, foremost in medicine and astronomy and in their shared border region astrology, has been traced back long in history by many authors from various perspectives, e.g., Bernal [5], Høyrup [23], and Kline [26].

Globally speaking, they all agree on three mathematization tendencies:

(1) The progress in the individual sciences makes work on ever more complicated problems possible and necessary.
(2) This accumulation of problems and data demands conscious, planned, and economic procedures in the individual sciences, i.e., an increased emphasis on questions of methodology.
(3) Finally, this increased emphasis on questions of methodology is as a rule associated with the tendency of mathematization.

All of this applies generally. In detail, we find many various pictures. In his *Groundplat of Sciences and Artes, Mathematicall* of 1570 [15], the English alchemist, astrologer, and mathematician John Dee, the first man to defend the Copernican theory in Britain and a consultant on navigation, pointed out, in best Aristotelian tradition, that it is necessary in the evaluation of mathematization to pay strict attention to the specific characteristics of the application area in question. He postulated a dichotomy between the *Principall* side, pure mathematics, and the *Deriuative* side, i.e., applied mathematics and mathematization. He then classified the applications of pure mathematics according to objects treated:

- *Ascending Application in thinges Supernaturall, eternall and Diuine*,
- *In thinges Mathematicall: Without farther Applications*,

and finally, on the lowest and most vulgar plane in the Aristotelian scheme

- *Descending Application in thinges Naturall: both Substantiall & Accidentall, Visible & Inuisible & c.*.

Now that history has excluded matters divine from mathematics, we can with some justification ask whether later generations may regard with equal amusement and astonishment the fact that in our time there are a large number of professional mathematicians, who are completely satisfied with spending their entire lives working in the second, inner mathematical, level and who persistently refuse to descend to vulgar applications.

The panorama of the individual sciences and the role that mathematics had to play in them was perfectly clear for John Dee. In our time the matter is somewhat more complex. In this review, I cannot point out a geodetically perfect picture of today's landscape of mathematization. I must treat the matter rather summarily. A summary treatment may have the advantage that in comparison among the mathematization progress in three sciences below (physics, medicine and economics), common problems on one hand and special features of mathematization on the other hand can be seen more clearly.

In the following, I shall "ascend" from the study of dead nature in *physics*, the field which has the highest degree of mathematization on any chosen scale, both quantitatively and qualitatively, over the investigation of living matter in *medicine*, the field where one might expect the greatest mathematization advances in our century, to the treatment of financial issues and decision making for commerce and production in *economics*, a field of questionable scientific state, that, beyond well founded actuary estimations, lacks unambiguous results and convincing clear perspectives regarding mathematization.

2. PHYSICS

The intimate connection between mathematics and physics makes it difficult to determine the theoretical relevance of mathematics and obscures the boundary between genuinely physical thought and observation on one side and the characteristically mathematical contribution on the other side. Recall Hilbert's perception of probability theory as a chapter of physics in his famous 6th Problem [22]:

6. Mathematical Treatment of the Axioms of Physics. The investigations on the foundations of geometry suggest the problem: *To treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.*

To say it mildly, as Gnedenko did in his comments to the Russian edition of 1969: *Today this viewpoint* (to consider probability theory as a chapter of physics) *is no longer so common as it was around the turn of the century, since the independent mathematical content of the theory of probabilities has sufficiently clearly showed since then...* With hindsight and in view of the still challenging foundational problems of quantum mechanics, however, we may accept that parts of mathematics and physics can be interlaced in a non-separable way.

Another famous example of that inextricable interlacement is provided by the Peierls-Frisch memorandum of 1940 to the British Government: suggested by the codiscoverer of fission Otto Frisch, the physicist Rudolph Peierls, like Frisch a refugee in Britain, made the decisive feasibility calculation that not tons (as - happily - erroneously estimated by Heisenberg in the service of the Nazis) but only about 1 kg (later corrected to 6 kg) of the pure fissile isotope U_{235} would be needed to make the atomic bomb. Was it mathematics or physics? It may be worth mentioning that

Peierls was full professor at the University of Birmingham since 1937 and became joint head of *mathematics* there, [17]. Theoretical physics in Britain is often in mathematics. As a matter of fact, physics in our sense did not exist as a single science before the nineteenth century. There were well-defined *experimental physics* comprising heat, magnetism, electricity and colour, leaving mechanics in mathematics, see [23, p. 493].

In spite of that intermingling, physics can provide a filter for our review, a ready system of categories to distinguish different use of mathematics in different modelling situations. Perhaps, the situation can be best compared with the role of physics in general education: after all, physics appears as the model of mathematization: there is no physics without mathematics - and, as a matter of fact, learning of mathematics is most easy in a physics context: calculation by letters; the various concepts of a function (table, graph, operation) and its derivatives and anti-derivatives; differential equations; the concept of observational errors and the corresponding estimations and tests of hypotheses; Brownian movements; all these concepts can be explained context-free or in other contexts (where some of the concepts actually originated), but they become clearest in the ideally simple applications of physics, which are sufficiently complicated to see the superiority of the mathematization as compared to feelings, qualitative arguments, discussions, convictions, imagination - but simple enough to get through.

2.1. **Variety of modelling purposes.** It may be helpful to distinguish the following modelling purposes:

2.1.1. *Production of data, model based measurements.* Clearly, the public associates the value of mathematical modelling foremost to its *predictive* power, e.g., in numerical weather prediction, and its *prescriptive* power, e.g., in the design of the internal ballistics of the hydrogen bomb; more flattering to mathematicians, the *explanatory* power of mathematization and its contribution to *theory development* yield the highest reputation within the field. However, to the progress of physics, the *descriptive* role, i.e., supporting model-based measurements in the laboratory, is – as hitherto – the most decisive contribution of mathematics. Visco-elastic constants and phase transition processes, e.g., of glasses and other soft materials can not be measured directly. For high precision in the critical region, one measures electric currents through a "dancing" piezoelectric disc with fixed potential and varying frequency. In this case, solving mathematical equations from the fields of electro-dynamics and thermo-elasticity becomes mandatory for the design of the experiments and the interpretation of the data. In popular terms, one may speak of a *mathematical microscope*, in technical terms of a *transducer* that becomes useful as soon as we understand the underlying mathematical equations.

2.1.2. *Simulation.* Once a model is found and verified and the system's parameters are estimated for one domain, one has the hope of doing computer calculations to predict what new experiments in new domains (new materials, new temperatures etc) should see. Rightly, one has given that type of calculations a special name of honour, *computer simulations*: as a rule, it requires to run the process on a computer or a network of computers under quite sophisticated conditions: typically, the problem is to bring the small distances and time intervals of well-understood molecular dynamics up to reasonable macroscopic scales, either by aggregation or by Monte Carlo methods – as demonstrated by Buffon's needle casting for the numerical approximation of π.

One should be aware that the word "simulation" has, for good and bad, a connation derived from NASA's space simulators and Nintendo's war games and juke boxes. Animations and other advanced computer simulations can display an impressive beauty and convincing power. That beauty, however, is often their dark side: simulations can show a deceptive similarity with true observations, so in computational fluid dynamics when the numerical solution of the Bernoulli equations, i.e., the linearization of the Navier-Stokes equations for laminar flow displays eddies characteristic for the non-linear flow. The eddies do not originate from real energy loss due to friction and viscosity but from hardly controllable hardware and software properties, e.g., the chopping of digits, thus providing a *magic realism*, as coined by Abbott and Larsen [1]. In numerical simulation, like in mathematical statistics, results which fit our expectations too nicely, must awake our vigilance instead of being taken as confirmation.

2.1.3. *Prediction.* As shown in the preceding subsection, there is no sharp boundary between description and prediction. However, the quality criteria for predictions are quite simple: do things develop and show up as predicted? So, for high precision astrology and longitudinal determination in deep-sea shipping the astronomical tables of planetary movement, based on the outdated and falsified Ptolemaic system (the *Resolved Alfonsine Tables*) and only modestly corrected in the *Prutenic Tables* of 1551 were, until the middle of the 17s century, rightly considered as more reliable than Kepler's heliocentric *Rudolphine Tables*, as long they were more precise - no matter on what basis, see, e.g., Steele [37, p. 128].

Almost unnoticed, we have had a similar revolution in weather prediction in recent years: the (i) analogy methods of identifying a similar looking weather situation in the weather card archives to base the extrapolation on it were replaced by almost pure (ii) numerical methods to derive the prediction solely from the thermodynamic and hydrodynamic basic equations and conservation laws, applied to initial conditions extracted from the observation grid. "Almost" because the uncertainty of the interpolation of the grid and the high sensitivity of the evolution equations to initial conditions obliges to repeated runs with small perturbations and human inspection and selection of the most "probable" outcome like in (i). That yields sharp estimates about the certainty of the prediction for a range of up to 10 days. In nine of ten cases, the predictions are surprisingly reliable and would have been impossible to obtain by traditional methods. However, a 10% failure rate would be considered unacceptable, e.g., in industrial quality control.

In elementary particle physics, the coincidence of predictions with measurements is impressive, but also disturbing. I quote from Smolin [36, pp. 12-13]:

Twelve particles and four forces are all we need to explain everything in the known world. We also understand very well the basic physics of these particles and forces. This understanding is expressed in terms of a theory that accounts for all these particles and all of the forces except for gravity. It's called *the standard model of elementary-particle physics* -or the standard model for short. ... Anything we want to compute in this theory we can, and it results in a finite number. In the more than thirty years since it was formulated, many predictions made by this theory have been checked experimentally. In each and every case, the theory has been confirmed.

The standard model was formulated in the early 1970s. Except for the discovery that neutrinos have mass, it has not required adjustment since. So why wasn't physics over by 1975? What remained to be done?

For all its usefulness, the standard model has a big problem: It has a long list of adjustable constants. ...

We feel pushed back to the pre-Keplerian, pre-Galilean and pre-Newtonian cosmology built on ad-hoc assumptions, displaying clever and deceptive mathematics-based similarity between observations and calculations – and ready to fall at any time because the basic assumptions are not explained.

Perhaps the word *deceptive* is inappropriate when speaking of description, simulation and prediction: for these tasks, *similarity* can rightly be considered as the highest value obtainable, as long one stays in a basically familiar context. From a semiotic angle, the very similarity must have a meaning and is indicating something; from a practical angle, questions regarding the epistemological status can often be discarded as metaphysical exaggerations: who cares about the theoretical or ad-hoc basis of a time schedule in public transportation – as long as the train goes on time!

2.1.4. *Control.* The prescriptive power of mathematization deserves a more critical examination. In physics and engineering we may distinguish between the (a) feasibility, the (b) efficiency, and the (c) safety of a design. A *design* can be an object like an airplane or a circuit diagram for a chip, an instrument like a digital thermometer, TV set, GPS receiver or pacemaker, or a regulated process like a feed-back regulation of the heat in a building, the control of a power station or the precise steering of a radiation canon in breast cancer therapy. Mathematics has its firm footing for testing (a) in thought experiments, estimations of process parameters, simulations and solving equations. For testing (b), a huge inventory is available of mathematical quality control and optimisation procedures, e.g., by variation of key parameters.

It seems to me, however, that (c), i.e., safety questions provide the greatest mathematical challenges. They appear differently in (i) experience-based, (ii) science-based and (iii) science-integrated design. In (i), mathematics enters mostly in the certification of the correctness of the design copy and the quality test of the performance. In (ii), well-established models and procedures have to be modified and re-calculated for a specific application. Experienced physicists and engineers, however, seldom trust their calculations and adaptations. Too many parameters may be unknown and pop-up later: Therefore, in traditional railroad construction, a small bridge was easily calculated and built, but then photogrammetrically checked when removing the support constructions. A clash of more than δ_{crit} required re-building. Similarly, even the most carefully calculated chemical reactors and other containers under pressure and heat have their prescribed "Soll-Bruchstelle" (supposed line of fracture) in case that something is going wrong.

The transition from (ii) to (iii) is the most challenging: very seldom one introduces a radically new design in the physics laboratory or engineering endeavour. But there are systems where all components and functions can be tested separately but the system as a whole can only be tested *in situ*: a new design of a Diesel ship engine; a car, air plane or space craft; a new concept in cryptography. In all these cases, one is tempted to look and even to advocate for mathematical proofs of the safe function according to specification. Unfortunately, in most cases these "proofs" belong rather to the field of fiction than to rigorous mathematics. For an interesting discussion on "proofs" in cryptography (a little remote from physics) see the debate between Koblitz and opponents in [27] and follow-ups in the *Notices of the American Mathematical Society*.

An additional disturbing aspect of science-integrated technology development is the danger of a loss of transparency. Personally, I must admit, I'm grateful for most black-box systems. I have no reason to complain when something in my computer is hidden for my eyes, as long everything functions as it shall or can easily be re-tuned. However, for the neighborhood of a chemical plant (and the reputation of the company) it may be better not to automatize everything but to keep some aspects of the control non-mathematized and in the hands of the service crew to avoid de-qualification and to keep the crew able to handle non-predictable situations.

A last important aspect of the prescriptive power of the mathematization is its formatting power for thought structure and social behaviour. It seems that there is not so much to do about it besides being aware of the effects.

2.1.5. *Explain phenomena.* Perhaps the noblest role of mathematical concepts in physics is to explain phenomena. Einstein did it when *reducing* the heat conduction to molecular diffusion, starting from the formal analogy of Fick's Law with the cross section of Brownian motion. He did it also when *generalizing* the Newtonian mechanics into the special relativity of constant light velocity and again when *unifying* forces and curvature in general relativity.

Roughly speaking, mathematical models can serve physics by reducing new phenomena to established principles; as heuristic devices for suitable generalizations and extensions; and as "a conceptual scheme in which the insights ... fit together" (C. Rovelli). Further below we shall return to the last aspect – the unification hope.

Physics history has not always attributed the best credentials to explaining phenomena by abstract constructions. It has discarded the concept of a ghost for perfect explanation of midnight noise in old castles; the concept of ether for explaining the finite light velocity; the phlogiston for burning and reduction processes, the Ptolemaic epicycles for planetary motion. It will be interesting to see in the years to come whether, e.g., the mathematically advanced String Theory or the recent Connes-Marcolli reformulation of the Standard Model in terms of spectral triples will undergo the same fate.

2.1.6. *Theory development.* Finally, what has been the role of mathematical concepts and mathematical beauty for the very theory development in physics? One example is John Bernoulli's purely aesthetic confirmation of Galilean fall law $s = g/2\, t^2$ among a couple of candidates as being the only one providing the same equation (shape) for his brachistochrone and Huygens' tautochrone, [6, p. 395]:

Before I end I must voice once more the admiration that I feel for the unexpected identity of Huygens' tautochrone and my brachystochrone. I consider it especially remarkable that this coincidence can take place only under the hypothesis of Galilei, so that we even obtain from this a proof of its correctness. Nature always tends to act in the simplest way, and so it here lets one curve serve two different functions, while under any other hypothesis we should need two curves.

Another, more prominent example is the lasting triumph of Maxwell's equations: a world of radically new applications were streaming out of the beauty and simplicity of the equations of electro-magnetic waves!

However, not every mathematical, theoretical and empirical accumulation leads to theory development. Immediately after discovering the high-speed rotation of the Earth around its own axis, a spindle shape of the Earth was suggested and an infinitesimal tapering towards the North pole confirmed in geodetic measurements around Paris. Afterwards, careful control measurements of the gravitation at the North Cap and at the Equator suggested the opposite, namely an ellipsoid shape with flattened poles. Ingenious mathematical mechanics provided a rigorous reason for that. Gauss and his collaborator Listing, however, found something different in their control. They called the shape *gleichsam wellenförmig* and dropped the idea of a theoretically satisfactory description. Since then we speak of a *Geoid.*

2.2. **"The trouble with physics".** That is the title of an interesting and well-informed polemic by Lee Smolin against String Theory and present main stream physics at large. He notices a *stagnation*in physics, *so much promise, so little fulfillment* [36, p. 313], a predominance of anti-foundational spirit and contempt for visions, partly related to the mathematization paradigm of the 1970s, according to Smolin: *Shut up and calculate.*

It seems to me that Smolin, basically, may be right. Børge Jessen, the Copenhagen mathematician and close collaborator of Harald Bohr once suggested to distinguish in sciences and mathematics between periods of *expansion* and periods of *consolidation.* Clearly physics had a consolidation period in the first half of the 20s century with relativity and quantum mechanics. The same may be true for biology with the momentous triumph of the DNA disclosure around 1950, while, to me, the mathematics of that period is characterized by almost chaotic expansion in thousands of directions. Following that way of looking, mathematics of the second half of the 20s century is characterized by enormous consolidation, combining so disparate fields like partial differential equations and topology in index theory, integral geometry and probability in point processes, number theory, statistical mechanics and cryptography, etc. etc. A true period of consolidation for mathematics, while - at least from outside - one can have the impression that physics and biology of the second half of the 20s century were characterized merely by expansion, new measurements, new effects - and almost total absence of consolidation or, at least failures and vanity of all trials in that direction.

Indeed, there have been impressive successes in recent physics, in spite of the absence of substantial theoretical progress in physics: perhaps the most spectacular and for applications most important discovery has been the High Temperature Superconducting (HTS) property of various ceramic materials by Bednorz and Müller - seemingly without mathematical or theoretical efforts but only by systematic combinatorial variation of experiments - in the tradition of the old alchemists, [4].

The remarkable advances in fluid dynamics, weather prediction, oceanography, climatic modelling are mainly related to new observations and advances in computer power while the equations have been studied long before.

Nevertheless, I noticed a turn to theory among young experimental physicists in recent years, partly related to investigating the *energy landscapes* in material sciences, partly to the re-discovery of the *interpretational* difficulties of quantum mechanics in recent quantum optics.

2.3. **Theory – model – experiment.** Physics offers an extremely useful practical distinction between *theory*, *model* and *experiments.* From his deep insight in astronomy, computing, linguistics and psychology, Peter Naur ridicules such distinctions as "metaphysical exaggeration", e.g., in [33]. He may be right. We certainly should not exaggerate the distinction. In this review, however, the distinction helps to focus on differences of the role of mathematics in doing science.

2.3.1. *First principles.* By definition, the very core of modelling is mathematics. Moreover, if alone by the stochastic character of observations, but also due to the need to understand the mathematics of all transducers involved in measurements, mathematics has its firm stand with experiments. First principles, however, have a different status: they do not earn their authority from the elegance of being mathematically wrapped, but from the almost infinite repetition of similar and, as well disparate observations connected to the same principle(s). In the first principles, mathematics and physics meet almost at eye level: first principles are also *established* - like mathematics, and are only marginally questioned. To me, the problem with the pretended eternal authority of first principles is that new cosmological work indicates that the laws of nature may also have undergone some development; that there might have "survived" some evolutionary relicts; and that we better should be prepared to be confronted, e.g., under extreme experimental conditions, with phenomena and relations which fall out of the range of accredited first principles. The canonical candidate for such a relict is the Higgs particle, whether already observed or not, see Holger Bech Nielsen's contributions, e.g., on the Quantum Gravity Assessment Workshop 2008, http://QuantumGravity.ruc.dk.

2.3.2. *Towards a taxonomy of models.* Not necessarily for the credibility of mathematical models, but for the way to check the range of credibility, the following taxonomy of models may be extremely useful.

The Closing Round Table of the International Congress of Mathematicians (Madrid, August 22-29, 2006) was devoted to the topic *Are pure and applied mathematics drifting apart?* As panellist, Yuri Manin subdivided the mathematization, i.e., the way mathematics can tell us something about the external world, into three modes of functioning (similarly Bohle, Booss and Jensen 1983, [7], see also [9]):

An *(ad-hoc, empirically based) mathematical model* "describes a certain range of phenomena, qualitatively or quantitatively, but feels uneasy pretending to be something more." Manin gives two examples for the predictive power of such models, Ptolemy's model of epicycles describing planetary motions of about 150 BCE, and the standard model of around 1960 describing the interaction of elementary particles, besides legions of ad-hoc models which hide lack of understanding behind a more or less elaborated mathematical formalism of organizing available data.

A *mathematically formulated theory* is distinguished from an ad-hoc model primarily by its "higher aspirations. A theory, so to speak, is an aristocratic model." Theoretically substantiated models, such as Newton's mechanics, are not necessarily more precise than ad-hoc models; the coding of experience in the form of a theory, however, allows a more flexible use of the model, since its embedding in a theory universe permits a theoretical check of at least some of its assumptions. A theoretical assessment of the precision and of possible deviations of the model can be based on the underlying theory.

A *mathematical metaphor* postulates that "some complex range of phenomena might be compared to a mathematical construction". As an example, Manin mentions artificial intelligence with its "very complex systems which are processing information because we have constructed them, and we are trying to compare them with the human brain, which we do not understand very well – we do not understand almost at all. So at the moment it is a very interesting mathematical metaphor, and what it allows us to do mostly is to sort of cut out our wrong assumptions. If we start comparing them with some very well-known reality, it turns out that they would not work."

Clearly, Manin noted the deceptive formal similarity of the three ways of mathematization which are radically different with respect to their empirical foundation and scientific status. He expressed concern about the lack of distinction and how that may "influence our value systems". In the words of [9, p. 73]:

Well founded applied mathematics generates prestige which is inappropriately generalized to support these quite different applications. The clarity and precision of the mathematical derivations here are in sharp contrast to the uncertainty of the underlying relations assumed. In fact, similarity of the mathematical formalism involved tends to mask the differences in the scientific extra-mathematical status, in the credibility of the conclusions and in appropriate ways of checking assumptions and results... Mathematization can – and therein lays its success – make existing

rationality transparent; mathematization cannot introduce rationality to a system where it is absent... or compensate for a deficit of knowledge.
Asked whether the last 30 years of mathematics' consolidation raise the chance of consolidation also in phenomenologically and metaphorically expanding sciences, Manin hesitated to use such simplistic terms. He recalled the notion of Kolmogorov complexity of a piece of information, which is, roughly speaking,

the length of the shortest programme, which can be then used to generate this piece of information... Classical laws of physics – such phantastic laws as Newton's law of gravity and Einstein's equations – are extremely short programmes to generate a lot of descriptions of real physical world situations. I am not at all sure that Kolmogorov's complexity of data that were uncovered by, say, genetics in the human genome project, or even modern cosmology data ... is sufficiently small that they can be really grasped by the human mind.

3. MEDICINE

3.1. **The special place of medicine.** Physicists of our time like to date the physics' beginning back to Galileo Galilei and his translation of measurable times and distances on a skew plane into an abstract fall law. Before Galilei - and long time after him, the methodological scientific status of what we would call mechanical physics was quite low as compared with medicine. Physics was a purely empirical subject. It was about precise series of observations and quantitative extrapolations. It was the way to predict planetary positions, in particular eclipse times, the content of silver in compounds, or the manpower required to lift a given weight with given weight arm. It was accompanied and mixed up with all kinds of speculations about the spirits and ghosts at work. We can easily see the continuity of results, of observations and calculations from Kepler and Newton to our time. However, we can hardly recognize anything in their thinking about physics, in the way they connected physics with cosmic music or alchemy.

3.1.1. *The maturity of medicine.* Contrary to that, from the rich ancient literature preserved, see Diepgen [16], Kudlien [28] and, in particular, Jürss [24, 312–315], we can see that the mind set in Greek medicine already from the fifth century BCE was *ours*: instead of the partition (familiar from earlier and shaman medicine and similar to the mind set preserved, as seen above, in physics until recent times) into an empirical-rational branch (healing wounds) and a religious-magic branch (cure inner diseases), a physiological concept emerged which focused on the patient as an individual organism within a population, with organs, liquids and tissue, subjected to environmental and dietetic influences and, in principle, open for unconfined investigation of functions, causal relations and the processive course of diseases. In Hippocratic medicine, we meet for the first time the visible endeavour after a rational surmounting of all problems related to body events.

With a shake of the head, we may read of Greek emphasis and speculations about the body's four liquids or other strange things, like when we recall today the verdict of the medical profession 60 years ago against drinking water after doing sports and under diarrhoea or their blind trust in antibiotics, not considering resistance aspects at all. Contrary to physics, we have no continuity of results in medicine, but, also contrary to physics, we have an outspoken continuity in mind set: no ghosts, no metaphysical spirits are permitted to enter our explanations, diagnoses, prevention, curation and palliation.

To me, medicine is a science that is characterized by maturity in thinking - but, until now, of ephemeral quality of results. There are astonishing progresses in the mathematization of biology and medicine of the last decades. There are good reasons to expect that medicine will become the most important field of mathematics application - as physics has been in the last centuries.

3.1.2. *Hopeless aspects of medicine mathematization.* Before proceeding with this review, three warnings shall be presented. The first warning is taken from Jean le Rond D'Alembert, Discours préliminaire to the Encyclopédie, [2, page vii]:

It must be confessed, however, that geometers sometimes abuse this application of algebra to physics. Lacking appropriate experiments as a basis for their calculations, they permit themselves to use hypotheses which are most convenient, to be sure, but often very far removed from what really exists in Nature. Some have tried to reduce even the art of curing to calculations, and the human body, that most complicated machine, has been treated by our algebraic doctors as if it were the simplest or the easiest one to reduce to its component parts. It is a curious thing to see these authors solve with the stroke of a pen problems of hydraulics and statics capable of occupying the greatest geometers for a whole lifetime. As for us who are wiser or more timid, let us be content to view most of these calculations and vague suppositions as intellectual games to which Nature is not obliged to conform, and let us conclude that the single true method of philosophizing as physical scientists consists either in the application of mathematical analysis to experiments, or in observation alone, enlightened by the spirit of method, aided sometimes by conjectures when they can furnish some insights, but rigidly dissociated from any arbitrary hypotheses.

A second warning against exaggerated expectations regarding the mathematization of biology and medicine comes from the suspicion that many of the organs, organelles and, e.g., DNA sequence regions are *evolutionary relicts* without any active function and with the only meaning to confuse the observer and the modeller.

A third warning relates to the conflict between *reductionist* and *holistic* approaches. We have seen a similar conflict in physics when discussing the chances vs. vanity of, e.g., the unifying approaches dating back to Einstein and re-actualized in various approaches to quantum gravity. For medicine it seems clear, that a strictly reductionist program is mandatory when we wish to replace ad-hoc assumptions by reference to first principles. On the other side, there is no doubt that most body functions and processes involve many parts of a cell, organ, and organism at the same time. Understandably, the slogan of holistic *Systems Biology* has become very popular and great expectations are attached to it. Clearly, both programs will disclose new and interesting facts and features and offer mathematicians rich fields to work on. To me, however, the most promising approach will be somewhere in between. Perhaps *focussed* systems biology will prove to be able to hit the wall and make a hole in the wall, i.e., a breakthrough. That is a medicine and biology discarding many surely relevant aspects, lengths, distances, and focussing on a restricted, but multi-level and multi-scale set of processes by applying a wide range of mathematical methods and first physics principles. For an example see below Paragraph 3.2.3.

3.2. **Populations, organisms, organs, cells, DNA.** In this report, we shall try to separate these levels wherever possible. On each level, quite different types of mathematical methods are applied and quite different types of results obtained.

3.2.1. *Populations.* Epidemiological studies about the distribution of disease in human populations and the factors determining that distribution, have shown to be useful in public health and preventive medicine, in particular when not only the interference was based on statistical methods but also the design of the data collection and the control of the reliability of the reported data.

Perhaps the most important result of mathematical epidemiology has been the statistical proof of the health risks of smoking - in spite of the great statistician Ronald A. Fisher's insistence that excess health problems and mortality of smokers might have two other clever explanations, besides the common, namely that subconscious feeling of early death might be the main reason of the desire to smoke or that a predisposition to lung cancer and arterial diseases are genetically confounded with the desire to smoke, see [18] and recognized and ridiculed the evidence as we would with the observed correlation between the occurrence of storks and the frequency of birth in the Mark Brandenburg in the 1920s (German children are told that the stork brings the babies in the same way as Anglo-Saxonic babies are found under gooseberry bushes). It must be acknowledged that the work of R.A. Fisher and his colleagues on correlation and regression analysis, goodness of fit, sampling and statistical error in the first half of the 20th century had given epidemiology new techniques of measurement and also undermined erroneous certainties. However, it remains strange that Fisher in his polemic did not disclose his role as advisor to the Tobacco Manufacturers Standing Committee nor admitted that a less libertarian world view would support the statistical results and health recommendations by Sir Richard Doll, the discoverer of the smoking correlation.

While main stream epidemiology has its greatest successes in identifying environmental and behavioural factors, *genetic epidemiology* impresses by identifying hereditary factors, in particular providing large panels of the most relevant single nucleotide polymorphisms (SNPs) associating in selected diseases. Recently, e.g., a SNP whole genome scan has been carried out on 1599 patients with type 2 diabetes and 1503 control subjects matched from Finland and Sweden, mapping loci for various elements of metabolic syndrome, see also Groop et al. [21]. While genotyping now begins to become rather straight forward, it remains often unclear how to translate the new information into theoretical analysis of the disease and to design new strategies for early diagnosis (perhaps not at all always desirable) and treatment, see also below Paragraph 3.2.3.

Another branch of epidemiology are the SIR models for *infectious diseases*. For teaching qualitative reasoning and numerical analysis of coupled non-linear differential equation, the modelling of Susceptibles, Infectives and Removed, respectively, Resistents have been a beloved dull for decades: a toy of pure mathematics and applied terminology like a sheep in wolf's clothing.

The power of SIR models became evident, however, in controversies, e.g., about vaccination schemes against Rubella, where the models could predict that non-vaccination would be more safe than non-compulsory vaccination below the critical level of coverage and Greece had to pay for not listening to the predictions, [3, pp. 77-79, 102-110, 152f, 167f, 188-199]. Modified SRI models have also proved applicable for estimating various factors for the spread (respectively, control) of Methicillin Resistant *Staphylococcus aurea* (MRSA): the role of antibiotic consumption, the role of isolation and other sanitary procedures in hospitals, the comparative weights of entrance and exit screening to keep the community pool down, etc., see [12].

It seems, that long-time oscillations of many other infectious diseases like the spread of the HIV virus, the measles or the chicken flu still challenge the mathematicians while not many colleagues believe that the short-time behaviour of these diseases really can be caught by these deterministic models in the presence of continuing permutations.

3.2.2. *Organisms and organs.* To give one (outstanding) example, we refer to the modelling of the cardio-vascular system, [34]. As discussed above in paragraph 2.1.3, computer supported modelling can provide the most beautiful animations. However, modelling can have other goals: explaining strange, but well-established phenomena (e.g., that the forces exercised by the heart are slightly below the forces measured by the blood stream); supporting heart surgery and implantations; and tuning, e.g., anaesthesia simulators for the training of anaesthesiologists. For these goals, the reliability of the modelling's basic assumptions has shown to be decisive. Compared with the most smashing animations, modelling the cardio-vascular system like a sewage system with elastic walls may sound primitive. However, it gains its reliability by its firm foundation on first principles.

3.2.3. *Cells.* The mathematization of the cell has many levels and many scales. To give an example, I shall describe an evolving - focussed - systems biology of regulated exocytosis in pancreatic β-cells, mostly based on [11]. These cells are responsible for the appropriate insulin secretion. Insufficient mass or function of these cells characterize Type 1 and Type 2 *diabetes mellitus* (T1DM, T2DM). Similar secretion processes happen in nerve cells. However, characteristic times for insulin secretion are between 5 and 30 minutes, while the secretion of neurotransmitters is in the millisecond range. Moreover, the length of a β-cell is hardly exceeding 3000 nanometres (nm), while nerve cells have characteristic lengths in the cm and meter range. So, processes in β-cells are more easily to observe than processes in nerve cells, but they are basically comparable.

It seems that comprehensive research on β-cell function and mass has been seriously hampered in 80 years because of the high efficiency of the symptomatic treatment

of T1DM and T2DM by insulin injection. Recent advances - and promises - of nanotechnology suggest the following radically new research agenda, to be executed first on cell lines, then on cell tissue of selected rodents, finally on living human cells:
1. Synthesize magneto-luminescent nanoparticles; develop a precisely working electric device, which is able to generate a properly behaving electromagnetic field; measure cytoskeletal viscosity and detect the interaction with organelles and actin filaments by optical tracking of the forced movement of the nanoparticles. Difficulties to overcome: protect against protein adsorption by suitable coating of the particles and determine the field strength necessary to distinguish the forced movement from the underlying Brownian motion.
2. Synthesize luminescent nanoparticles with after-glow property (extended duration of luminescence and separation of excitation and light emission); dope the nanoparticles with suitable antigens and attach them on selected organelles to track intracellular dynamics, e.g., of the insulin granules.
3. Develop a multipurpose sensor chip and measure all electric phenomena (varying potential over the plasma membrane, the bursts of Ca^{++} ion oscillations, and changing impedances on the surface of the plasma membrane for precise chronical order of relevant secretion events.
4. Describe the details of the bilayer membrane-granule fusion event (with the hard numerical problems of the meso scale, largely exceeding the well-functioning scales of molecular dynamics).
5. Connect the preceding dynamic and geometric data with reaction-diffusion data.
Connect the preceding dynamic and geometric data with genetic data.
Develop clinical and pharmaceutical applications:

- Quality control of transplants for T1DM patients.
- Testing drug components for β-cell repair.
- Testing nanotoxicity and drug components for various cell types.
- Early in-vivo diagnosis by enhanced gastroscopy.
- Develop mild forms of gene therapy for patients with over-expressed major type 2-diabetes gene TCF7L2 by targeting short interfering RNA sequences (siRNAs) to the β-cells, leading to degradation of excess mRNA transcript. (This strategy may be difficult to implement, due to the degradation of free RNA in the blood and the risk of off-target effects.)

We shall not go in the mathematical details of the involved compartment models, free boundary theory, reaction-diffusion equations, data analysis etc. Summing up, the mathematization can help design relevant experiments and support the determination of basic parameters. Mathematical modelling and simulation can point to *possible* new phenomena or new relations which have to be confirmed and investigated more thoroughly in the laboratory. However, at the present state of our knowledge about cell and cell membrane processes, one must doubt whether making predictions *in silico* can really replace experiments *in vivo* or *in vitro*.

4. ECONOMICS

While a holistic unifying view in physics has a smell of vanity (in spite of efforts in quantum gravity) and the time hardly has come for a holistic all-embracing systems biology programme in medicine, a rational point of departure for economics, in particular under the present crisis, can only be a *systems view*.

On one side we have the relief of breaking the growth curve which has been depriving future generations of non-renewable resources. In some parts of the world we can say "Enough is enough". We can choose a conscious way of living, we can replace purely quantitative scales for the quality of life, like GNP, money, work places, hospital beds and spending power by other social and individual values like health, peace, prosperity, happiness. We can look to Cuba or other model countries who have shown how to live, e.g., with dramatically restricted energy consumption - if a lot more should go wrong.

On the other side we must worry that the crisis, like the Great Depression of 1929-1945, see [25] will deepen the differences between rich and poor within and between countries, and can foster unprecedented political and military catastrophes.

Many scientific subjects are challenged to give explanation and advise in the present crisis. We would be grateful for relief in understanding what is going on, and qualified warning would be needed against leaving the required decisions to the isolated world of so-called *decision makers* or *chief advisors* and *chief economists*. Perhaps not mathematical economics is needed (see, however, the recent ex-post analyses, e.g., [14]), but rather psychology which is proficient in how individuals and masses experience crises, or philosophy which should be proficient at the relation between property and lack of sense of responsibility.

What can a mathematician add to the avalanche of crisis comments in the literature? Where are the pitfalls and sins and where the chances of mathematical modelling in economics and finance under the present crisis?

4.1. **Praise and criticism of greed and accountability.** In laborious analysis of hundred thousands of figures and tables, the classic socialist thinkers, culminating with Friedrich Engels and Karl Marx, have shown that capitalism builds on, supports and develops *Rechenhaftigkeit – accountability*, and that abstract greed, greed for more money, greed for more profit, is the most powerful and progressive aspect of capitalism. They advocated for another society where decisions, human relations, development should not be driven by greed but by conscious and debated choices. That makes meaning. But clinging to capitalism, though preferably without greed, is logical non-sense.

One does not need relatively advanced macro-economical predictive models to see that greed, while driving economic growth and economisation of (many, not all) resources, generates contradictions. It generates abundance of free capital in the presence of unsatisfied consumption wishes and deepens the social divide. Under this perspective, it was not at all unreasonable by the US-American Federal Bank to admit the application of *subprime* loans of questionable security: many young men of US-American lower middle class were risking health and life in Iraq and Afghanistan.

As Alan Greenspan confessed: under the given circumstances, the subprime loans were necessary. It *gave more people a stake in the future of our country and boded well for the cohesion of the nation* in a country at war, [20, Chapter 11]!

In many comments, the so-called *neo-classical economic equilibrium theory* is blamed for having closed the eyes of politicians and bankers in front of the upcoming crisis. Mathematical economics can, in deed, be blamed for many misleading concepts, statements and predictions. Mathematical models have had, e.g., their share in moving the focus from basic contradictions in the sectors of production and consumption (private daily and luxury requirements and state and military expenditures, i.e., *Main Street*) over into the sphere of circulation (*Wall Street*) proliferating for much too long the impression that the financial crisis only was a problem for banks and pension funds. No doubt, floods of money and signs are more easy to quantify and mathematize than the state and development of productive forces and production relations.

However, mathematical models can not really be blamed for the crisis. The crisis was there, it was open, it was visible, and one reacted correctly according to the conditions of capitalism - and made so the crisis to deepen and to propagate into a broader crisis.

Now, the system-immanent rational continues when greed and state power are merged in an unprecedented way and huge and almost free self-service tables are provided for poisoned capital on the verge of bankruptcy. Once again, it's clearly the correct answer within a context which is considered as fixed. Consequently, one must be afraid that these help packages will further deepen the crisis, at least for a time to come until the financial markets will regain a certain transparency and will show their force to clean-up the mess - with sacrifices of yet unknown magnitude and composition to be offered to the gods.

4.2. **Hedging and the fight for security.** Mathematical modelling in finance and economics has not only greed, growth and profit as variables to predict and optimize, but also the security of investments, the safety of an portfolio of private investors, hedge funds and pension funds. Actually, mathematicians, actuaries and mathematically trained economists can make calculations that appear valid, credible and trustworthy. Thirty years ago, there were only two mathematicians employed by finance in the whole of Denmark. Since a couple of years, often more than half of a year of graduates in math and physics end up there.

Credit evaluation and fixing prices of new financial creations is so intricate that parts of the most modern mathematics, e.g., martingales and heat equation, are challenged. In *normal* situations, the calculations hold. That's the way they are *calibrated*. These calculations have had, admittedly, their part in conjuring false security. It is ironical that the mathematicians were employed for enhancing the security of investments by modelling. However, by making the trade with options and other financial derivates more just, more reliable, more easy, we have also created a machine that can magnify and worsen a developing crisis by market automatics.

4.3. **Soft selection instead of control of the irrational?** In discussions about the crisis one often hears the word *complexity*, *uncertainty*, *control* and *chaos*. In mathematics we know many very simple, even deterministic systems that are absolutely unpredictable as long they are permitted to follow their way. Take, e.g., the double pendulum: it can turn somersaults. Small, practically invisible differences decide whether there comes one somersault more. Unpredictable? Yes. Uncontrollable? Yes - unless we put a sufficiently narrow box around the pendulum. Then no further somersaults will come. Perhaps such a box should be built for our economic systems as well, sooner or later.

As discussed in the previous chapters, mathematical models are able, perhaps like all science, to predict concrete events (like eclipses and election results) and consequences of performed operations (e.g., when we put on the light at the switch). Mathematics and science, however, can do that in ideally-simple cases only. In complex cases, we will be happy if we at least can point to necessary changes, confinements, regulations to create a predictable and adjustable system. Instead of the possible vain goals of controlling an irrational system we may consider to soften our system of goal functions. In the long run, soft selection and redesign of priorities and ways of cooperation, as explained, e.g., in [19], will be the most efficient way to peace and prosperity. To me, that is the only proved theorem of this paper.

5. GENERAL TRENDS OF MATHEMATIZATION AND MODELLING

5.1. **Deep divide.** Regarding the power and the value of mathematization, there is a deep moral divide both within the mathematics community and the public.

On the one side, we have the outspoken science and math optimism of outstanding thinkers Henri Poincaré's *Nature not only suggests to us problems, she suggests their solution*; David Hilbert's *Wir müssen wissen; wir werden wissen. - We must know; we will know.* of his Speech in Königsberg in 1930, now on his tomb in Göttingen; or Bertolt Brecht's vision of mathematical accountability in *Die Tage der Kommune* [13] of 1945: "Das ist die Kommune, das ist die Wissenschaft, das neue Jahrtausend... - That is the Commune, that is the science, the new millennium..."). We have astonishing evidence that many mathematization concepts either appear to us as *natural* and *a-priori*, or they use to emerge as clear over time. We have the power and validity of extremely simple concepts, as in *dimension analysis*, *consistency requirements* and *gauge invariance* of mathematical physics. Progressive movements emphasize science and education in liberation movements and developing countries. Humanitarian organizations (like WHO and UNICEF) preach science and technology optimism in confronting mass poverty and epidemics.

On the other side, deep limitation layers of science and mathematical thinking have been dogged up by Kurt Gödel's *Incompleteness Theorem* for sufficiently rich arithmetic systems, Andrei N. Kolmogorov's *Complexity Theory*, and Niels Bohr's notion of *Complementarity*. Incomprehensibility and lack of regularity continue to hamper trustworthy mathematization. Peter Lax [29, p. 142] writes of the *profound* *mystery of fluids*, though recognizing that different approaches lead to remarkable coincidence results, supporting reliability.

The abstruseness of the mathematical triumphs of the hydrogen bomb is common place. The wide spread trust on superiority and invincibility, based on mathematical war technology like high precision bombing, has proved to be even more vicious for warriors and victims than the immediate physical impact of the very math-based weaponry, recently also in Iraq and Afghanistan.

In between the two extremes, we have the optimistic scepticism of Eugene Wigner's *unreasonable effectiveness of mathematics*, but also Jacob Schwartz's verdict against the *pernicious influence of mathematics on science* and Albert Einstein's demand for *finding the central questions* against the dominance of the beautiful and the difficult.

5.2. **Charles Sanders Peirce's semiotic view.** From the times of Niels Bohr, many physicists, mathematicians and biologists have been attentive to philosophical aspects of our doing. Most of us are convinced that the frontier situation of our research can point to aspects of some philosophical relevance - if only the professional philosophers would take the necessary time to become familiar with our thinking. Seldom, however, we read something of the philosophers which can inspire us.

The US-American philosopher Charles Sanders Peirce (1839-1914) is an admirable exception. In his semiotics and pragmaticist (he avoids the word "pragmatic") thinking, he provides a wealth of ideas, spread over an immense life work. It seems to me, that many of his ideas, comments and concepts can shed light on the why and how of mathematization. Here I shall only refer some thoughts of Peirce's *The Fixation of Belief* of 1877, see [35].

My fascination of Peirce's text is, in particular, based on the following observations which may appear trivial, but are necessary to repeat many times for the new-modeller:

1. For good and bad, we all are equipped with innate (or spontaneous) orientation, sometimes to exploit, sometimes to subdue. Our innate orientation is similar to the habits of animals in familiar neighborhood. We all are "logical machines".
2. However, inborn logic is not sufficient in foreign (new) situations. For such situations, we need methods how to fixate our beliefs. Peirce distinguishes four different methods. All four have mathematical aspects and are common in mathematical modelling.

Tenacity: is our strength not becoming confused, not to be blown away by unfounded arguments, superficial objections, misleading examples, though sometimes keeping our ears locked too long.

Authority: of well-established theories and results is what we tend to believe and have to stick at. We will seldom drop a mastered approach in favour of something new and unproved.

Discussion: can hardly help to overcome a belief built on tenacity or authority.

Consequences: have to be investigated in all modelling. At the end of the day, they decide whether we become convinced of the validity of our approach (Peirce's Pragmaticist Maxim).

3. The main tool of modelling (i.e., the fixation of belief by mathematical arguments) is the transformation of symbols (signals, observations, *segments of reality*) into a new set of symbols (mathematical equations, *models* and *descriptions*). The advantage for the modeller, for the person to interpret the signs, is that signs which are hard or humid and difficult to collect in one hand can be replaced by signs which we can write and manipulate.
4. The common mapping cycle *reality* → *model* → *validation* is misleading. The quality of a mathematical model is not how similar it is to the segment of reality under consideration, but whether it provides a flexible and goal-oriented approach, opening for doubts and indicating ways for the removal of doubts (later trivialized by Popper's falsification claim). More precisely, Peirce claims

Be aware of differences between different approaches!

Try to distinguish different goals (different priorities) of modelling as precise as possible!

Investigate whether different goals are mutually compatible, i.e., can be reached simultaneously!

Behave realistic! Don't ask: *How well does the model reflect a given segment of the world*? Better ask: *Does this model of a given segment of the world support the wanted and possibly wider activities / goals better than other models*?

5.3. **Integral geometry, for ever.** Perhaps a brief summary of the history of *integral geometry*, from Cavalieri's Principle to Harmonic Analysis, Characteristic Classes, Point Processes, and Stochastic Geometry could illustrate some of Peirce's points and explain the origin and development of far-reaching mathematical concepts, the power of mathematization, from gauge theoretic physics to the counting of brain cells, and the continuing and deepening limitations of mathematization over urging problems, be it the limitations of mathematical concepts like *homotopy invariance* (when, e.g., confronted with spectral invariants) or limitations due to peculiarities of the subject domain. There is more to discuss.

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