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CONCIRCULAR CURVATURE TENSOR IN CONTACT METRIC MANIFOLDS

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Abstract. The paper considers (κ, μ)-manifolds that satisfy $Z(\xi, X) \cdot S = 0$, where Z stands for the concircular curvature tensor and S for Ricci tensor. A clasification of $N(\kappa)$-contact metric manifolds satisfying $Z(\xi, X) \cdot Z = 0$, $Z(\xi, X) \cdot R = 0$, $R(\xi, X) \cdot Z = 0$ is proposed, where R is the curvature tensor. There are some applications to concircularly symmetric $N(\kappa)$-contact metric manifolds or manifolds possessing non-vanishing recurrent concircular curvature tensor.

§1. INTRODUCTION

Semisymmetric spaces have been investigated by E. Cartan, they generalize the symmetric spaces ($\nabla R = 0$). A Riemannian manifold M is said to be semisymmetric, if its curvature tensor R satisfies

$$R(X, Y) \cdot R = 0, \quad X, Y \in TM,$$

where $R(X, Y)$ acts on R as a derivative. A Riemannian manifold M is Ricci-semisymmetric (sometimes Ricci-semiparallel), if its Ricci tensor S is semisymmetric, that is, its curvature tensor R satisfies $R(X, Y) \cdot S = 0$, $X, Y \in TM$, where $R(X, Y)$ acts on S as a derivative. Ricci-semisymmetric Riemannian manifolds are natural generalizations of symmetric spaces ($\nabla R = 0$), Einstein spaces, semisymmetric spaces ($R(X, Y) \cdot R = 0$) and Ricci-symmetric Riemannian manifolds ($\nabla S = 0$) (see [13] for more details). In [14], V. Mirzoyan proved a general structure theorem for Ricci-semisymmetric manifolds asserting that a Riemannian manifold is Ricci-semisymmetric if and only if it is 2-dimensional or an Einstein space or a semi-Einstein space or a local product of such spaces. In contact geometry, S. Tanno [20] showed that a semisymmetric K-contact manifold M^{2n+1} is locally isometric to the unit sphere $S^{2n+1}(1)$. He also proved that for a K-contact manifold M the following four conditions are equivalent :

(a) M is an Einstein manifold,

(b) M possesses parallel Ricci tensor (that is, M is Ricci-symmetric),

(c) M satisfies $R(X,Y)\cdot S=0$ (that is, M is Ricci-semisymmetric) and

(d) M satisfies $R(\xi,X)\cdot S=0$, where ξ is the structure vector field.

Since a Sasakian manifold is always a K-contact manifold, this result is valid for Sasakian manifolds. Thus, a Ricci-semisymmetric Sasakian manifold is an Einstein manifold. This generalizes a result of M. Okumura [17], which states that any Ricci-symmetric Sasakian manifold is an Einstein manifold.

We remark that a contact metric manifold of constant curvature is necessarily a Sasakian manifold of constant curvature $+1$ or is 3-dimensional and flat (see [16] or [5], pp. 98-99). A contact metric manifold M^{2n+1} satisfying $R(X,Y)\xi=0$, where ξ is the characteristic vector field of the contact structure, is locally isometric to $E^{n+1}\times S^n(4)$ for $n>1$ and flat in dimension 3 (see [5], p. 101).

D. Perrone studied contact metric manifolds satisfying $R(\xi,X)\cdot R=0$ in [19], where he showed that under additional assumptions the manifold is either Sasakian (and of constant curvature $+1$) or $R(X,\xi)\xi=0$. B. J. Papantoniou [18] showed, that a semisymmetric contact metric manifold M^{2n+1} with ξ belonging to the (κ,μ)-nullity distribution is locally isometric to $S^{2n+1}(1)$ or to $E^{n+1}\times S^n(4)$. Both Perrone and Papantoniou also studied manifolds satisfying $R(\xi,X)\cdot S=0$, where S denotes the Ricci tensor. Perrone shows that if ξ belongs to the κ-nullity distribution and if $R(\xi,X)\cdot S=0$, then the contact metric manifold is locally isometric to $E^{n+1}\times S^n(4)$ or is Sasakian-Einstein. The author [22] improved the results of B. J. Papantoniou [18] for (κ,μ)-manifolds satisfying $R(\xi,X)\cdot S=0$.

In [2], C. Baikoussis and T. Koufogiorgos showed that if ξ belongs to the κ-nullity distribution and if $R(\xi,X)\cdot C=0$, C being the Weyl conformal curvature tensor, the contact metric manifold M^{2n+1} is locally isometric to $S^{2n+1}(1)$ or to $E^{n+1}\times S^n(4)$. This generalizes a result of Chaki and Tarafdar [10] that a Sasakian manifold M^{2n+1} such that $R(\xi,X)\cdot C=0$ is locally isometric to $S^{2n+1}(1)$. Moreover, in [15] Murathan and Yildiz studied (κ,μ)-manifolds satisfying $C(\xi,X)\cdot S=0$.

The present paper is mainly based on the joint work of the author with Professor D. E. Blair and Dr. Jeong-Sik Kim (see [8], [23]). Section 2 contains necessary details about contact metric manifolds. In section 3, we give a brief account of (κ,μ)-manifolds and also present two results. In section 4, we explain the notion of $\mathcal{D}$-homothetic deformation and construct a key example for later use. We give a brief introduction to concircular curvature tensor in section 5. In section 6, we give an example of a non-Sasakian η-Einstein manifold, present a structure theorem for non-Sasakian η-Einstein manifolds, and give classifications of (κ, μ)-manifolds satisfying $Z(\xi, X) \cdot S = 0$. In section 7, we classify $N(\kappa)$-contact metric manifolds satisfying $Z(\xi, X) \cdot Z = 0$, $Z(\xi, X) \cdot R = 0$, $R(\xi, X) \cdot Z = 0$ and point at some applications to $N(\kappa)$-contact metric manifolds, which are concircularly symmetric or are with non-vanishing recurrent concircular curvature tensor. In the end an open problem is proposed.

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§2. CONTACT METRIC MANIFOLDS

An odd-dimensional manifold M^{2n+1} is said to admit an **almost contact structure**, sometimes called a (φ, ξ, η)**-structure**, if it admits a tensor field φ of type $(1,1)$, a vector field ξ, and a 1-form η satisfying

$$\varphi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \varphi\xi = 0, \quad \eta \circ \varphi = 0. \tag{1}$$

The first and one of the remaining three relations in (1) imply the other two relations in (1). Let g be a compatible Riemannian metric with (φ, ξ, η), that is,

$$g(X, Y) = g(\varphi X, \varphi Y) + \eta(X)\eta(Y)$$

for all $X, Y \in TM$. Then, M becomes an **almost contact metric manifold** equipped with an almost contact metric structure (φ, ξ, η, g). An almost contact metric structure becomes a **contact metric structure**, if

$$g(X, \varphi Y) = d\eta(X, Y), \quad X, Y \in TM.$$

The 1-form η is then a contact form and ξ is its characteristic vector field. An almost contact metric manifold is Sasakian if and only if

$$(\nabla_X \varphi)Y = g(X, Y)\xi - \eta(Y)X, \quad X, Y \in TM,$$

where ∇ is Levi-Civita connection. A contact metric manifold M is Sasakian if and only if the curvature tensor R satisfies

$$R(X, Y)\xi = R_0(X, Y)\xi, \quad X, Y \in TM, \tag{2}$$

where $R_0(X,Y)U = g(Y,U)X - g(X,U)Y, \quad X,Y,U \in TM$.
A contact metric manifold is called a **K-contact manifold**, if the characteristic vector field ξ is a Killing vector field. An almost contact metric manifold is K-contact if and only if $\nabla\xi = -\varphi$. A K-contact manifold is a contact metric manifold, while the converse is true if $h = 0$, where $2h$ is the Lie derivative of φ in the characteristic direction ξ. A Sasakian manifold is always a K-contact manifold. A 3-dimensional K-contact manifold is a Sasakian manifold. Thus a 3-dimensional contact metric manifold is a Sasakian manifold if and only if $h = 0$. For more details see [5].

§3. (κ,μ)-MANIFOLDS

It is well known that the tangent sphere bundle of a flat Riemannian manifold admits a contact metric structure satisfying $R(X,Y)\xi = 0$ [3]. On the other hand, as we have noted (see (2)), on a Sasakian manifold $R(X,Y)\xi = \eta(Y)X - \eta(X)Y$. As a generalization of both $R(X,Y)\xi = 0$ and the Sasakian case ; D. Blair, T. Koufogiorgos and B. J. Papantoniou [6] considered the **(κ,μ)-nullity condition** on a contact metric manifold. The **(κ,μ)-nullity distribution** $N(\kappa,\mu)$ ([6], [18]) of a contact metric manifold M is defined by

$$N(\kappa,\mu) : p \longmapsto N_p(\kappa,\mu) = \{U \in T_pM \mid R(X,Y)U = (\kappa I + \mu h)R_0(X;Y)U\}$$

for all $X,Y \in TM$, where $(\kappa,\mu) \in \mathbb{R}^2$. A contact metric manifold M^{2n+1} with $\xi \in N(\kappa,\mu)$ is called a **(κ,μ)-manifold**. In particular on a (κ,μ)-manifold, we have

$$R(X,Y)\xi = \kappa(\eta(Y)X - \eta(X)Y) + \mu(\eta(Y)hX - \eta(X)hY).$$

On a (κ,μ)-manifold $\kappa \leq 1$. If $\kappa = 1$, the structure is Sasakian ($h = 0$ and μ is not determined) and if $\kappa < 1$, the (κ,μ)-nullity condition determines the curvature of M^{2n+1} completely [6]. In fact, among (κ,μ)-manifolds, the subclasses of Sasakian manifolds, K-contact manifolds coincide, and are described by $\kappa = 1$ and $h = 0$. Moreover, we have $Q\xi = 2n\kappa\xi, \quad h^2 = (\kappa-1)\varphi^2$, where Q is Ricci operator. If $\mu = 0$, the (κ,μ)-nullity distribution $N(\kappa,\mu)$ is reduced to the **κ-nullity distribution** $N(\kappa)$ [21], where the **κ-nullity distribution** $N(\kappa)$ of a Riemannian manifold M is defined by

$$N(\kappa) : p \longmapsto N_p(\kappa) = \{U \in T_pM \mid R(X,Y)U = \kappa R_0(X,Y)U\},$$

where κ is a constant. If $\xi \in N(\kappa)$, then a contact metric manifold M we call **$N(\kappa)$-contact metric manifold**. If $\kappa = 1$, an $N(\kappa)$-contact metric manifold is Sasakian and if $\kappa = 0$, an $N(\kappa)$-contact metric manifold is locally isometric to $E^{n+1} \times S^n(4)$. In [1], where $N(\kappa)$-contact metric manifolds were studied, it was shown that $\kappa < 1$

implies that the scalar curvature $r = 2n(2n-2+\kappa)$. For more detail we refer to [1] and [6].

The standard contact metric structure on the tangent sphere bundle T_1M satisfies the (κ,μ)-nullity condition if and only if the base manifold M is of constant curvature. In particular if M has constant curvature c, then $\kappa = c(2-c)$ and $\mu = -2c$. To end the section, we reproduce the following results :

Theorem 3.1 [23]. *A Ricci flat (κ,μ)-manifold is necessarily flat and 3-dimensional.*

Theorem 3.2 [23]. *A non-Sasakian Einstein (κ,μ)-manifold is necessarily 3-dimensional and flat.*

Theorem 3.2 is a generalization of the following

Theorem 3.3 [21] *If an $N(\kappa)$-contact metric manifold of dimension ≥ 5 is Einstein, then it is necessarily Sasakian.*

§4. $\mathcal{D}$–HOMOTHETIC DEFORMATION

For a given contact metric structure (φ,ξ,η,g), a **$\mathcal{D}$-homothetic deformation** is the structure defined by

$$\bar{\eta} = a\eta, \quad \bar{\xi} = \frac{1}{a}\xi, \quad \bar{\varphi} = \varphi, \quad \bar{g} = ag + a(a-1)\eta\otimes\eta,$$

where a is a positive constant. It preserves the contact metric, K-contact, Sasakian or strongly pseudo-convex CR properties, but destroys the relations like $R(X,Y)\xi = 0$ or $R(X,Y)\xi = \kappa(\eta(Y)X - \eta(X)Y)$. However, the form of the (κ,μ)-nullity condition is preserved under a $\mathcal{D}$-homothetic deformation with

$$\bar{\kappa} = \frac{\kappa + a^2 - 1}{a^2}, \quad \bar{\mu} = \frac{\mu + 2a - 2}{a}.$$

Given a non-Sasakian (κ,μ)-manifold M, Boeckx [9] introduced an invariant

$$I_M = \frac{1-\frac{\mu}{2}}{\sqrt{1-\kappa}}$$

and showed that for two non-Sasakian (κ,μ)-manifolds $(M_i,\varphi_i,\xi_i,\eta_i,g_i)$, $i = 1,2$ we have $I_{M_1} = I_{M_2}$ if and only if up to a $\mathcal{D}$-homothetic deformation, the two manifolds are locally isometric as contact metric manifolds. Thus we know all non-Sasakian (κ,μ)-manifolds locally as soon as we have for every odd dimension $2n+1$ and for every possible value of the invariant I, one (κ,μ)-manifold (M,φ,ξ,η,g) with $I_M = I$. For $I > -1$ such examples can be built using the standard contact metric structure on the tangent sphere bundle of a manifold of constant curvature c, where we have $I = \frac{1+c}{|1-c|}$.

Boeckx also gives a Lie algebra construction for any odd dimension and values of $I \leq -1$. Using that invariant, we now construct an example of a $(2n+1)$-dimensional $N\left(1-\frac{1}{n}\right)$-contact metric manifold, $n > 1$.

Example 4.1. Since the Boeckx invariant for a $(1-\frac{1}{n},0)$-manifold is $\sqrt{n} > -1$, we consider the tangent sphere bundle of an $(n+1)$-dimensional manifold of constant curvature c so chosen that the resulting $\mathcal{D}$-homothetic deformation will be a $(1-\frac{1}{n},0)$-manifold. That is, for $\kappa = c(2-c)$ and $\mu = -2c$ we solve $1-\frac{1}{n} = \frac{\kappa+a^2-1}{a^2}$, $0 = \frac{\mu+2a-2}{a}$ for a and c. For the values $c = \frac{(\sqrt{n}\pm1)^2}{n-1}$, $a = 1+c$, we obtain a $N\left(1-\frac{1}{n}\right)$-contact metric manifold. Example 4.1 will be used in Theorems 6.3, 7.1 and 7.2.

§5. CONCIRCULAR CURVATURE TENSOR

A transformation of an n-dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle, is called a **concircular transformation** ([12], [24]). (A geodesic circle is a curve in M whose first curvature is constant and whose second curvature is identical zero.) A concircular transformation is always a conformal transformation ([12]). Thus, the geometry of concircular transformations, that is, the concircular geometry, is a generalization of inversive geometry in the sense that the change of metric is more general than that induced by a circle preserving diffeomorphism (see also [4]). An interesting invariant of a concircular transformation is the **concircular curvature tensor** Z ([24], [25]) :

$$Z = R - \frac{r}{n(n-1)}R_0,$$

where R is the curvature tensor and r is the scalar curvature. From the form of the concircular curvature tensor we conclude that Riemannian manifolds with vanishing concircular curvature tensor are of constant curvature. A necessary and sufficient condition that a Riemannian manifold can be reduced to a Euclidean space by a concircular transformation is that its concircular curvature tensor vanishes.

§6. (κ,μ)-MANIFOLDS WITH $Z(\xi,X)\cdot S=0$

A contact metric manifold M is said to be **η-Einstein** (see [17] or [5], p. 105), if the Ricci tensor S satisfies

$$S = ag + b\eta\otimes\eta, \tag{3}$$

where a and b are some smooth functions on the manifold. In particular if $b=0$, then M becomes an **Einstein manifold**. In dimensions ≥ 5 it is known, that for any η-Einstein K-contact manifold, a and b are constants [20].

We note that a non-Sasakian (κ,μ)-manifold M^{2n+1} is η-Einstein if and only if $\mu = -2(n-1)$. In particular, a 3-dimensional contact metric manifold is η-Einstein if and only if it is an $N(\kappa)$-contact metric manifold [7]. More precisely, in a 3-dimensional

$N(\kappa)$-contact metric manifold

$$S = \left(\frac{r}{2} - \kappa\right) g + \left(3\kappa - \frac{r}{2}\right) \eta \otimes \eta. \tag{4}$$

Example 6.1. A contact metric manifold, obtained by a $\mathcal{D}$-homothetic deformation of the contact metric structure on the tangent sphere bundle of a Riemannian manifold M^{n+1} of constant curvature $\frac{n^2 \pm 2n+1}{n^2-1}$, is a non-Sasakian η-Einstein (κ, μ)-manifold. In a non-Sasakian η-Einstein (κ, μ)-manifold M^{2n+1}, we have

$$S = 2(n^2 - 1)g - 2(n^2 - n\kappa - 1)\eta \otimes \eta. \tag{5}$$

Theorem 6.1 [23]. *Let M^{2n+1} be a non-Sasakian η-Einstein (κ, μ)-manifold. Then the concircular curvature tensor Z satisfies $Z(\xi, X) \cdot S = 0$ if and only if M^{2n+1} is flat and 3-dimensional.*

We close this section with the following theorem.

Theorem 6.2 [23]. *Let M^{2n+1} be a (κ, μ)-manifold. The concircular curvature tensor Z satisfies $Z(\xi, X) \cdot S = 0$ if and only if one of the following conditions is satisfied :*

(a) M^{2n+1} is flat and 3-dimensional.

(b) M^{2n+1} is locally isometric to the Example 4.1.

(c) M^{2n+1} is an Einstein-Sasakian manifold.

§7. $N(\kappa)$-CONTACT METRIC MANIFOLDS SATISFYING $Z(\xi, X) \cdot Z = 0$

We now present a theorem in which Example 4.1 arises naturally in contrast to $E^{n+1} \times S^n(4)$, cf. Theorem 7.3 below.

Theorem 7.1 [8]. *A $(2n+1)$-dimensional $N(\kappa)$-contact metric manifold M satisfies $Z(\xi, X) \cdot Z = 0$, if and only if M is locally isometric to the sphere $S^{2n+1}(1)$, M is locally isometric to the Example 4.1 or M is 3-dimensional and flat.*

The following theorem is a corollary.

Theorem 7.2 [8]. *A $(2n+1)$-dimensional $N(\kappa)$-contact metric manifold M satisfies $Z(\xi, X) \cdot R = 0$, if and only if M is locally isometric to the sphere $S^{2n+1}(1)$, M is locally isometric to Example 4.1 or M is 3-dimensional and flat.*

On the other hand, reversing the order of Z and R gives the following result.

Theorem 7.3 [8]. *A $(2n+1)$-dimensional $N(\kappa)$-contact metric manifold M satisfies $R(\xi, X) \cdot Z = 0$, if and only if M is locally isometric to the sphere $S^{2n+1}(1)$ or to $E^{n+1} \times S^n(4)$.*

A Riemannian manifold is said to be **concircularly symmetric**, if the concircular curvature tensor Z is parallel, that is

$$\nabla Z = 0. \tag{6}$$

Theorem 7.4 [8]. *Let M^{2n+1} be a concircularly symmetric $N(\kappa)$-contact metric manifold. Then M is locally isometric to either $E^{n+1}(0) \times S^n(4)$ or the sphere $S^{2n+1}(1)$.*

Remark. We note that while Z is a concircular invariant, the connection ∇ is not and hence $\nabla Z = 0$ is not a concircular invariant. It can be interesting to study spaces, which are concircularly equivalent to a locally symmetric space.

If we assume that the concircular curvature tensor Z in an $N(\kappa)$-contact metric manifold M^{2n+1} is recurrent, that is

$$\nabla Z = \alpha \otimes Z, \tag{7}$$

where α is an everywhere non-vanishing 1-form, then we have the following theorem.

Theorem 7.5 [8]. *Let M^{2n+1} be an $N(\kappa)$-contact metric manifold with non-vanishing recurrent concircular curvature tensor. Then M^{2n+1} is locally isometric to $E^{n+1}(0) \times S^n(4)$.*

To conclude, we propose the following problem.

Problem. To classify (κ, μ)-manifolds under conditions $Z(\xi, X) \cdot Z = 0$, $Z(\xi, X) \cdot R = 0$, $R(\xi, X) \cdot Z = 0$, $\nabla Z = 0$, and $\nabla Z = \alpha \otimes Z$.

Резюме. В статье рассматриваются (κ, μ)-многообразия, удовлетворяющие $Z(\xi, X) \cdot S = 0$, где Z означает концикулярный тензор кривизны, а S – тензор Риччи. В работе предлагается классификация $N(\kappa)$-касательных метрических многообразий, удовлетворяющих $Z(\xi, X) \cdot Z = 0$, $Z(\xi, X) \cdot R = 0$, $R(\xi, X) \cdot Z = 0$, где R – тензор кривизны. Имеются несколько применений концикулярных симметрических $N(\kappa)$-касательных метрических многообразий или многообразий, обладающих ненулевым периодическим концикулярным тензором кривизны.

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