АСТРОФИЗИКА

ТОМ 47 ФЕВРАЛЬ, 2004 ВЫПУСК 1

УДК: 52-332

THE PARAMETRIZATION OF THE GROUND STATE AND ITS APPLICATIONS TO PLANETARY SYSTEMS

Tapan K.CHATTERJEE[1], V.B.MAGALINSKY[1,2]

Received 28 May 2003
Accepted 3 November 2003

Natural systems tend to minimize their energy. Hence an important problem in astrophysics is the parametrization of the ground state. In this context a quantum statistical approach is very useful. The problem of the variational approximation of the density matrix is extended towards a parametrization of the ground state. With an analogy to the semiclassical approach, a classical approach to the variational principle in the parametrization of the ground state is elucidated and its applications are discussed. We find that planetary systems tend to have circular orbits in an effort to attain the ground state. The results of this paper may be useful for the modern problem of detecting planets around bright stars.

Key words: *planetary systems*

1. *Introduction.* Since the (claimed) discovery of the first extra-solar planet or exoplanet (in 1995), the number of such bodies has grown rapidly (over 100), leading to the modern problem of detecting planets around bright stars and pulsars (e.g., [1]). An analysis of the distribution of orbital elements is already throwing light on their formation and evolution (e.g., [2]). Such that a study of the expected orbital trends of planetary systems is of vital importance.

As natural systems tend towards minimization of energy, hence a problem of paramount importance in astrophysics is to parameterize the ground state. In this context the quantum statistical methods are very convenient. The density matrix contains a lot of information (see, e.g., [3]). The problem of the approximation of the density matrix by a path integral over a conditional Wiener-Feynman measure was treated in previous papers [4-6]. The variational principle used in such problems and functionals are very useful in the parametrization of the ground state of a system. The aim is to formulate an analogue in the classical approach and determine the characteristics of the ground state. In this context, we extend the variational approach, in the semi-classical and classical domains and arrive at the parametrization of the ground state in the context of astronomical systems.

2. *Theory.* The variational method consists of the construction of an extremum of the path integral, using an extremum condition (e.g., [4,5]).

Along with the real system, we consider an auxiliary simulating system, which possess the same kinetic energy but different potential energy $V_0(y, \alpha)$, α being the variational parameter over which we extremise; (y corresponding to the degree of freedom). An alternative is to introduce the variational parameters in terms of a parametrization of the wave function for the ground state. Results of [6] indicate that in the semi-classical approach, a functional, S, provides information for the parametrization of the ground state. With a linear parametrization in the invariants of the flux, the projection of the functional S into the parameter space of the phase flux, is an extended one of action, in which the fluctuations of the ground state is taken into account; while its projection in configurational space is a parameterized one of the ground state.

An analogue of this method, in the classical approach, to parameterize the ground state, is elucidated. In classical approach the concept of extended space can be conceived by an example of the application of the variational principle to classical mechanics. We can consider the orbital motion of a binary in which the secondary, of unit mass, has a mass negligible compared to the primary. We can study the tidal effects on the secondary, due to primary, by considering the latter to be a static point, of mass M ($M \gg 1$). We can combine the parameter space of relative orbital motion and rotational motion of the secondary by considering an extended space of 6 dimensions. The functional, S (which gives information on the probability over the path), corresponds here to the total energy of the system. The extremum criterion is applied to the energy of the system, in terms of a softened length which is a function of the separation between the components and a scale length pertaining to the rotational motion of the secondary. (One can also visualize an analogy of the Lagrange variables with the internal variables of the secondary and the wave packet with the secondary. Just as the functional variables appear to be the center of the trajectories and the dispersion of the parameterized wave packet, the latter being reduced at the ends of the trajectories to zero width, the internal motion of the secondary can be considered to be an anisotropic change of scale, maintaining symmetry).

Let $\mathbf{r} = \mathbf{r}(t)$ be the position of the center of the mass of the protoplanet (seconday) of unit mass ($\ll M$), with respect to the parent star (primary), of mass M, considered to be a static point mass. Let the mass of the secondary be distributed about its center $\eta = 0$, here $\eta = [\eta_i]$, ($i = 1, 2, 3$), with density distribution $\phi(\eta, t)$; such that, its internal movement can be described by the velocity field $\mathbf{u}(\eta, t)$. The secondary can be described as a cloud of particles of dust and gas or other internal constituents. During its orbital motion, it changes its size and density, due to tidal forces.

The equation of continuity implies,

$$\partial_t \phi + \nabla_{\eta_l}(\phi u_l) = 0, \tag{1}$$

where η_l correspond to the coordinates of the volume element with respect to the center of mass of the secondary and

$$\int \phi\, d^3\eta_l = 1 = \text{const}. \quad (M >> 1), \tag{2}$$

$$\int \eta_l \phi\, d^3\eta_l = 0. \tag{3}$$

Which implies that the secondary maintains its symmetry and the evolution of the distribution function of its density, ϕ, reduces to an anisotropic, variable scale transformation, with three scale factors $D_l'(t)$, which can be imposed as

$$\phi(\eta_l, t) = J(\xi)\prod_1^3 [D_l'(t)]^{-1}, \tag{4}$$

with

$$\xi_l = \eta_l / D_l'. \tag{5}$$

Equations (2) and (3), imply that the form factor, $J(\xi)$, is normalized by

$$\int J d^3\xi = 1. \tag{6}$$

and

$$\int \xi_l J d^3\xi = 0. \tag{7}$$

We denote the radius of gyration of the secondary, corresponding to its moment of inertia about its axis of rotation by $D = |\mathbf{D}|$, which is given by,

$$D_l = \langle \xi^2 \rangle^{1/2} D_l'. \tag{8}$$

The internal velocity field of the secondary corresponding to its potential field is obtained from Equations (1) and (4) as,

$$u_l = \dot{D}_l \xi_l = (\dot{D}_l'/D_l')\eta_l \tag{9}$$

Thus considering the orbital motion and rotation of the secondary (but neglecting its internal thermal motion), we obtain its kinetic energy as,

$$K = (1/2)\int \phi(\dot{\mathbf{r}} + \mathbf{u})^2 d^3\eta = (1/2)\int (\dot{\mathbf{r}}^2 + \dot{\mathbf{D}}^2) d^3\eta \tag{10}$$

[using the Relations (4) to (9)].

Neglecting the interaction between the constituent particles of the secondary, the potential energy of the system reduces to the gravitational interaction between the primary and the secondary, given by,

$$\upsilon = -GM\int |\mathbf{r} + \eta|^{-1} \phi(\eta, t) d^3\eta \tag{11}$$

which can be represented in the form of a mean using Equation (4) as,

$$\upsilon = -GM\left\langle [(\mathbf{r} + \eta)^2]^{-1/2} \right\rangle \tag{12}$$

Nothing that υ is a convex function with respect to its argument $(\mathbf{r}+\eta)^2$, we can approximate υ from above by using the Jansen inequality, $\upsilon_{ap} \geq \upsilon_{ex}$, i.e.,

$$-GM\left\langle\left[(\mathbf{r}+\eta)^2\right]^{-1/2}\right\rangle \leq -GM\left[\left\langle(\mathbf{r}+\eta)^2\right\rangle\right]^{-1/2}, \tag{13}$$

where υ_{ex} and υ_{ap} denote the exact and approximate values of υ, respectively. Taking into account Relation (8), we obtain,

$$\upsilon_{ap} = -GM\left(r^2 + D^2\right)^{-1/2} \tag{14}$$

Unifying the vectors, $\mathbf{r}$ and $\mathbf{D}$, in extended space we obtain (the symbol $\oplus$ denoting an unification of space),

$$\mathbf{r} \oplus \mathbf{D} = \mathbf{R} \equiv [x_l, \eta_l]; \quad (l = 1, 2, 3). \tag{15}$$

The extended space can obviously be decomposed into two (mutually) orthogonal subspaces: one (external) containing the positions of the centers of mass of the primary and secondary, and the other (internal) containing the scale factors corresponding to the secondary, such that, in the extended space,

$$\mathbf{r} \cdot \mathbf{D} = 0, \quad r^2 + D^2 = R^2, \quad \dot{r}^2 + \dot{D}^2 = \dot{R}^2, \tag{16}$$

Such that (14) imply, $\upsilon_{ap} = -GM/R$.

From the integrals of motion deduced it is clear that the movement takes place in a *plane* (in the extended space), where we can introduce a polar coordinate system with the pole at the center of mass of the primary (the source of the external force). The polar radius $\mathbf{R}$ and the polar angle Θ are taken so that the (elliptical) trajectory of the secondary (with its gravity softened) can be expressed in the parametric form,

$$\mathbf{R} = \mathbf{P}/(1 + E\cos\Theta), \tag{17}$$

where E and P denote the eccentricity and semi-latus rectum of the orbit, respectively. The angular momentum integral can be expressed as,

$$L = R^2\,\dot{\Theta} \tag{18}$$

This leads to the energy approximation,

$$E_{ap} = K + [\upsilon_L]_{ap} = (1/2)\dot{R}^2 + \left[-(GM/R) + \left\{L^2/\left(2R^2\right)\right\}\right], \tag{19}$$

where υ_L is the effective potential and $[\upsilon_L]_{ap}$ its approximation.

This implies that the error of the approximation is minimum when E_{ex} has its minimum value; that is for the ground state. The system tends to the ground state for $\dot{R} = 0$, implying $R = R_0 = L^2/GM = \text{const}$; such that we have a *circular* Keplerian motion in the extended space.

Notice that if the size of the secondary is insignificant compared to the primary and hence to the instantaneous orbital separation between the two, $D << r$, then $R \approx r$ and $r \approx \text{const}$, for the ground state; and we have circular orbit in 3-dimensional space for the secondary, for the ground state.

Also, note that the general relationship $r^2 + D^2 = R^2 = L^2/GM = \text{const}$, implies that, as r increases, D decreases and vice versa. Thus the secondary is more extended when it is nearer to the primary and vice versa, compatible with the tidal theory.

3. *Applications and discussions.* Planetary systems are becoming quite frequent; but the orbital parameters for not many have been determined accurately. Also the components of the system must not be interacting; otherwise the tidal evolution will be masked by more intense interactions and we will not be able to segregate the effects of interchange of orbital and rotational energies in an effort of the system to attain the ground state. But after a violent evolution, characterized by interaction, the system can again attain a quiescent phase (e.g., for X-ray binaries); so we can study the tidal evolution in the quiescent phase and apply our theory.

Lineweaver et al. [7] and Grether&Lineweaver [8] plotted the mass-period distribution for nearly 100 expoplanets; they labeled a region in this diagram ("detected", containing 46 expoplanets) corresponding to the part where the Doppler technique has detected virtually all expoplanets (with period < 3 yrs) orbiting target stars (monitored for at least 3 yrs). They find that the average orbital eccentricity in this region $\langle e \rangle = 0.3$, which, according to our theory, indicates that the circularization process, due to energy minimization, is in progress. Jupiter-like planets are important since Jupiter is the dominant orbiting body in the Solar System and it lies in the most densely occupied region of the mass-period plane. They find that in the mentioned region of the mass-period distribution they can define a Jupiter-like sub-region (defined by orbital periods between the average period of the asteroid belt and Saturn and masses in the range roughly between the mass of Saturn and 3 times the mass of Jupiter); the average eccentricity of 5 expoplanets in this sub region is $\langle e \rangle = 0.2$, indicating that eccentricities may be decreasing as we approach this Jupiter-like sub-region. According to our theory, this can be interpreted as a progress in the circularization process towards energy minimization.

In the observational side there is now increasing evidence to support the claim that stars with planets tend to be metal rich; approximately 20% of metal rich main sequence stars have planets (e.g., [2,9]). There is also a significant positive correlation between the occurrence of planets and the iron content of their parent stars (e.g., [10,11]). Such that (near) solar type stars are favored to harbor planets. The relative orbit of Barnard's star (spectral type M) show a deviation from smooth track of about a few hundredths of an arc second, which show a roughly periodic structure. Analysis of these deviations by van de Kamp [12,13] show that the motion of Barnard's star can be explained in terms of two companions of masses ~

Jupiter's mass and half of Jupiter's mass in circular relative orbits about the star (with periods ~12 yrs and 20 yrs, respectively); though there is uncertainty about the observations of the outer companion, there is considerable confidence about the inner one. Our results indicate that this system is likely to be in its ground state. (However, there is a controversy (see e.g., [14]); but the periodic nature of the deviations from an analysis of several thousands of photographic plates support the results of van de Kamp).

However, as noticed, in most of the cases cited above the near circular orbit is found to be the best fit and not confirmed to be circular. Where the orbital eccentricity can be determined with remarkable accuracy is in the field of pulsar astronomy. Before a star evolves to a neutron star its environment is subjected to violent explosions and only under remotely conceivable conditions [15,16] can companions of planetary mass survice. On the other hand such companions may form (from the debris of a former companion destroyed or ejected) or be captured after the star attained the neutron state. Two planets are found to orbit the 6 millisecond pulsar 1257 + 12, (see, e.g., [17]). These planets have several times the mass of the Earth and nearly circular orbits, the eccentricity being 0.02 for both, with periods of 66.6 days and 98.2 days for the inner and outer planet, respectively. It is remotely conceivable that the planets are survivors from the pre-neutron star phase of the pulsar, not only for the violent scenario involved, but also because of the high spatial velocity of the pulsar (~290 km/s). The pulsar has a short period and small dipole field, characteristic of recycled pulsars; indicative of the fact that its progenitor was a binary system, implying it had a companion. Hence a natural scenario for the formation of the planets is from the debris of the pulsar's ex-companion star, destroyed gradually (see, e.g., [18]), or catastrophically (see, e.g., [19]), due to the explosions involved in the variable phase of the progenitor of the pulsar. Though most of this debris would accrete in an accretion disk around the neutron star, a small fraction has to be thrown outwards to conserve angular momentum, which becomes the source for planet formation; (see, e.g., [20]). Our results indicate that the planetary companions of the pulsar have circularized their orbits, in an effort to attain the ground state.

Multiple systems also seem to conform to our theory. In fact, the presence of binaries can induce disk instability under certain conditions, enhancing planet formation (e.g., [21]). For the double star 61 Cygni (spectral type K) [though to be a binary system with two components of comparable mass (~0.6 solar mass)], it was found that the distance between the two components vary periodically; a analysis of the path of these residuals by Diech [22] indicate that the system has an unseen companion of negligible mass (~4 Jupiter masses) in an orbit of eccentricity 0.2 (with a period ~6 yrs); (another companion of insignificant, outer to the mentioned one, of orbital period ~12 yrs, may be present); our results indicate that the unseen companion is likely to be close to its ground state.

Stars with more massive companions, like a binary with a brown dwarf component, also conform to our theory. A perturbation, with a period of 4.63 yrs, in the path of the star LHS 1047 (spectral type dM) was noted [24]; subsequently the companion was detected by the technique of one-dimensional infrared speckle interferometry. A least square analysis of the data indicate that the photometric orbit has an eccentricity of 0.095; the secondary is ~4 times lighter than the primary (mass of the primary = 0.17 ± 0.12 and secondary 0.06 ± 0.03 in solar mass units). In the system BD + 66°34 (dM spectral type) (thought to be a visual binary), it was found that the brighter component shows a large periodic perturbation (e.g., [25]). The orbit of the visual components was found by trial and error method. On the basis of the perturbation, the orbit of the unseen companion, about the brighter star and its mass was determined (~1% error in period and semi-major axis) and is found to be (~0.1 solar mass) ~4 times lower than brighter component (~0.4 solar mass) and its orbital eccentricity is 0.05. The system ζ Aquarii (dF spectral type) [thought to be a visual binary with components of comparable mass (~1.13 and 0.85 solar masses)], was found to display large and regular deviations from Keplerian motion, which can be attributed to an unseen third companion in orbit around the fainter component (e.g., [26]). The residuals of the relative orbit of the visual components showed a 25 yrs periodic variation, from which the elements of the orbit of the unseen component about the fainter component were derived; by means of alternate iterations, the sets of elements of both the orbits were refined, until the combination gave a minimum for the sum of the squared residuals. The mass of the unseem component is found to be (~0.28 solar mass) ~4 times lower than the brighter component and its orbital eccentricity is 0.20. Such that all these subsystems are close to the ground state.

It is a matter of great pleasure to thank all those with whom we had helpfull discussions; especially the organizers of the conferences "Path-Integrals from peV to TeV" (Florence, 1998) and IAU Symposium 219 "Stars as Suns: Activity, Evolution and Planets" (Sydney, 2003) and the referee, whose comments helped in improving the paper.

[1] Puebla University, Mexico, e-mail: chtapan@yahoo.com
[2] University of the Friendship of Peoples, Moscow, Russia

ПАРАМЕТРИЗАЦИЯ ОСНОВНОГО СОСТОЯНИЯ И ЕЕ ПРИМЕНЕНИЕ К ПЛАНЕТАРНЫМ СИСТЕМАМ

Т.К.ЧАТЕРДЖИ[1], В.Б.МАГАЛИНСКИЙ[1,2]

Естественные системы имеют тенденцию минимизировать свою энергию. Поэтому важной проблемой астрофизики является параметризация основного состояния. В этом контексте очень полезен квантовый статистический подход. Проблема вариационного приближения матрицы плотности распространяется на параметризацию основного состояния. По аналогии с квазиклассическим подходом, разъясняется классический подход к вариационному принципу параметризации основного состояния. Обсуждается его применение. Мы показываем, что для достижения основного состояния планетарные системы стремятся иметь круговые орбиты. Результаты настоящей работы могут быть полезными для современной проблемы обнаружения планет вокруг ярких звезд.

REFERENCES

1. *D.A.Fisher, P.B.Butler, G.W.Mercy, S.S.Vogt, G.W.Henry*, Astrophys. J., **590**, 1081, 2003.
2. *N.Santos, M.Mayor, S.Udry, D.Queloz, F.Pepe*, IAU Symp., **219**, "Stars as Suns: Activity, Evolution and Planets", in Press, 2003, Publ. Astron. Soc. Pacific.
3. *R.P.Feynman*, Statistical Mechanics (Benjamin, Reading, Mass.), 1972.
4. *V.B.Magalinsky*, Z. Eksp. Teor, Fiz., **99**, 184, 1991; - Sov. Phys. JETP, **72**, 103, 1991 (English translation).
5. *V.B.Magalinsky, T.K.Chatterjee*, Proceedings of Conf. "Path Integrals from peV to Tev", eds. R.Casalbouni et al., World Scientific Publ. Co., Singapore, p.505, 1999.
6. *V.B.Magalinsky, M.Hayasi, H.V.Mendoza*, J. Phys. Soc. Japan, **63**, 2930, 1994.
7. *C.H.Lineweaver, G.Grether, M.Hidas*, ASP Conf. Vol. **294**, "Scientific Frontiers in Research on Extrasolar Planets", eds. D.Deming, S.Seager, p.161, 2003, Publ. Astron. Soc. Pacific.
8. *G.Grether, C.H.Lineweaver*, IAU Symp. **219**, "Stars as Suns: Activity, Evolution and Planets", in Press, 2003, Publ. Astron. Soc. Pacific.
9. *D.A.Fischer, J.Valenti*, IAU Symp. **219**, "Stars as Suns: Activity, Evolution and Planets", in Press, 2003, Publ. Astron. Soc. Pacific.
10. *K.Sadakane*, IAU Symp. **219**, "Stars as Suns: Activity, Evolution and

Planets", in Press, 2003, Publ. Astron. Soc. Pacific.
11. *G.Israelian*, IAU Symp. **219**, "Stars as Suns: Activity, Evolution and Planets", in Press, 2003, Publ. Astron. Soc. Pacific.
12. *P.Van de Kamp*, Astron. J., **80**, 658, 1975.
13. *P.Van de Kamp*, Bull. American Astron. Soc., **14**, 627, 1982.
14. *R.S.Harrington*, *B.J.Harrington*, Mercury, **16**, 77, 1987.
15. *R.A.M.J.Wijers et al.*, Nature, **355**, 593, 1992.
16. *S.E.Thorsett*, *R.H.Dewey*, Astrophys. J. (L), **419**, L65, 1993.
17. *A.Wolszczan*, *D.A.Frail*, Nature, **355**, 145, 1992.
18. *M.Banit*, *M.A.Ruderman*, *J.Shaham*, *J.H.Applegate*, Astrophys. J., **415**, 779, 1993.
19. *E.S.Phinney*, *B.M.S.Hansen*, CALTECH, p.371, 1993.
20. *S.P.Ruden*, CALTECH, p.197, 1993.
21. *A.P.Boss*, Astrophys. J., **503**, 923, 1998.
22. *A.N.Deich*, Pis'ma Astron. Zh., **4**, 95, 1978; - Sov. Astron. Lett., **4**, 50, 1978, (English translation).
23. *W.J.Forrest*, *M.F.Skrutskie*, *M.Shure*, Astrophys. J. (L), **330**, L119, 1988.
24. *P.A.Ianna*, *J.R.Rohde*, *D.W.Mccarthy*, Astron. J., **95**, 1226, 1988.
25. *J.H.Hershey*, Astron. J., **78**, 935, 1973.
26. *O.Franz*, Astron. J., **63**, 329, 1958.